The discounted utility model

$Z = \{z_1, z_2, \ldots, z_m\}$ set of basic outcomes $T = \{0, 1, 2, \ldots, n\}$ a set of dates

$t = 0$ is now, $t = 1$ is one period from now …

(z, t) : **outcome *z* experienced at date *t***

Preferences over the set of dated outcomes: indexed by the date at which the preferences are being considered:

$\underbrace{(z,1)}_{\text{outcome } z \text{ is to be experienced at date } 1} \succ_0 (z',2)$ means: at date 0 z at time 1 is preferred to z' at time 2

RESTRICTION: $(z,t) \succsim_s (z',t')$ implies that $t \geq s$ $t' \geq$

from the point of view of date s

U_s utility function that represents the preferences at date *s*:

$$U_s(z,t) \geq U_s(z',t') \text{ if and only if } (z,t) \succsim_s (z',t')$$

When the preferences at time *s* are restricted to outcomes to be experienced at time *s* then simpler notation $u_s(z)$:

$$u_s(z) = U_s(z,s)$$

Call $u_s(z)$ the *instantaneous utility of z at time s.*

Begin with preferences at time 0 (the present): $\succsim_0$ represented by $U_0(\bullet)$.
The **discounted or exponential utility model** assumes that these preferences have the following form:

$$U_0(z,t) = \delta^t \underbrace{u_t(z)}_{= U_t(z,t)} \qquad (*)$$

δ = discount factor

$U_0(z,0) = \delta^0 u_0(z) = u_0(z)$

$\delta = \frac{1}{1+\rho}$ $\qquad \rho$ = discount rate

$\underbrace{(z,t)}_{} \succsim_0 (z',s)$ if and only if

$U_0(z,t) \geq U_0(z',s)$

$\shortparallel$

$\delta^t u_t(z) \geq \delta^s u_s(z')$

Example 1. z = take online yoga class, z' = take in-person yoga class

$$(z,1) \sim_0 (z',3)$$

If her preferences satisfy the discounted utility model then

$$\underbrace{U_0(z,1)}_{\delta^1 u_1(z)} = \underbrace{U_0(z',3)}_{\delta^3 u_3(z')}$$

Suppose that $\underline{u_1(z) = 4}$ and $\underline{u_3(z') = 6}$.

$\delta \cdot 4 = \delta^3 \cdot 6 \qquad \frac{4}{6} = \delta^2 \qquad \delta = \sqrt{\frac{4}{6}}$

1. Then what is her discount factor? $\rightarrow$ $= 0.8165$

2. What is her discount rate? (ρ | h)

$\delta = \frac{1}{1+\rho} \qquad \frac{1}{1+\rho} = 0.8165$

$\rho = 0.2247$

$$U_0(z,t) = \delta^t u_t(z)$$

Suppose you have a choice between $(z',0)$, $(z,0)$ and $(z,1)$

z' = do nothing and z = carry out a particular activity

$U_0(z',0) = \delta^0 u_0(z') = u_0(z')$

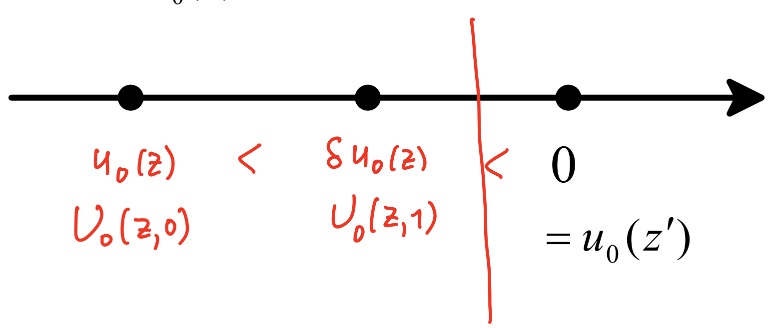
$U_0(z,0) = \delta^0 u_0(z) = u_0(z)$

$U_0(z,1) = \delta u_1(z) = \delta u_0(z)$

Suppose that $u_0(z) = u_1(z)$

Suppose that $\boxed{u_0(z') = 0}$ and $u_1(z) = u_0(z)$ so that $U_0(z,1) = \delta u_1(z)$

- $u_0(z) < \underbrace{0}_{=u_0(z')}$ z unpleasant activity

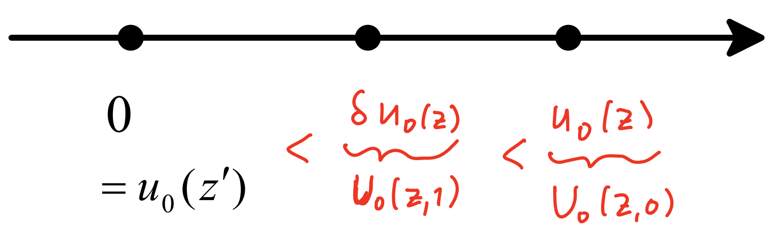


procrastination

- $u_0(z) > \underbrace{0}_{=u_0(z')}$ z pleasant activity

$U_0(z,1) = \delta u_1(z)$

$0 < \delta < 1$



↑ choose this

captures impatience

prefer $(z,0)$ to $(z,1)$

Ranking sequence of outcomes

EXAMPLE 2.

	Today	*Tomorrow*
date	0	1
Plan A	x	y
Plan B	y	x

Suppose: $\underbrace{u_0(x) = u_1(x) = 4}$ $\quad u_0(y) = u_1(y) = 6 \quad \delta = 0.8.$

	Today	*Tomorrow*
date	0	1
Plan A	4	6
Plan B	6	4

Extension of the discounted utility:

$$U_0(\text{Plan A}) = \delta^0 4 + \delta^1 6 = 4 + (0.8)6 = 8.8$$

$$U_0(\text{Plan B}) = \delta^0 6 + \delta^1 4 = 6 + (0.8)4 = 9.2$$

EXAMPLE 3.

date	0	1	2
Plan A	x	y	z
Plan B	y	z	x

$U_0(\text{Plan A}) =$ $\delta^0 u_0(x) + \delta^1 u_1(y) + \delta^2 u_2(z)$

$U_0(\text{Plan B}) =$ $\delta^0 u_0(y) + \delta^1 u_1(z) + \delta^2 u_2(x)$

Suppose $\begin{cases} \delta = 0.9, \\ u_0(x) = 0,\ u_1(y) = 4,\ u_2(z) = 2 \\ u_0(y) = 3,\ u_1(z) = 1,\ u_2(x) = 1 \end{cases}$, then

date	0	1	2
Plan A			
Plan B			

$U_0(\text{Plan A}) =$ 5.22 ← choose plan A

$U_0(\text{Plan B}) =$ 4.71