The discounted utility model

$Z = \{z_1, z_2, ..., z_m\}$ set of basic outcomes $T = \{0,1,2,...,n\}$ a set of dates

$t = 0$ is now, $t = 1$ is one period from now …

(z,t) : **outcome *z* experienced at date *t***

Preferences over the set of dated outcomes: indexed by the date at which the preferences are being considered:

$(z,1) \succ_0 (z',2)$ means:

RESTRICTION: $(z,t) \succsim_s (z',t')$ implies that

U_s utility function that represents the preferences at date *s*:

When the preferences at time *s* are restricted to outcomes to be experienced at time *s* then simpler notation $u_s(z)$:

$$u_s(z) =$$

Call $u_s(z)$ the *instantaneous utility of z at time s.*

Begin with preferences at time 0 (the present): $\succsim_0$ represented by $U_0(\bullet)$.
The **discounted or exponential utility model** assumes that these preferences have the following form:

(*)

$(z,t) \succsim_0 (z',s)$ if and only if

Example 1. z = take online yoga class, z' = take in-person yoga class

$$(z,1) \sim_0 (z',3)$$

If her preferences satisfy the discounted utility model then

Suppose that $u_1(z) = 4$ and $u_3(z') = 6$.

1. Then what is her discount factor?

2. What is her discount rate?

$$U_0(z,t) = \delta^t u_t(z)$$

Suppose you have a choice between $(z',0)$, $(z,0)$ and $(z,1)$

z' = do nothing and z = carry out a particular activity

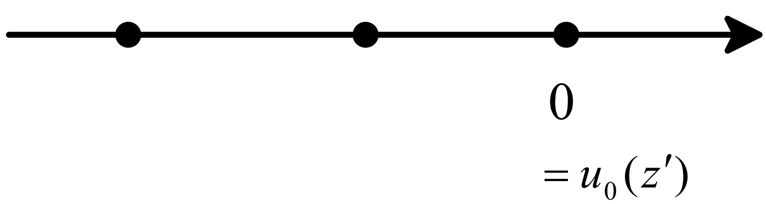
$U_0(z',0) =$

$U_0(z,0) =$

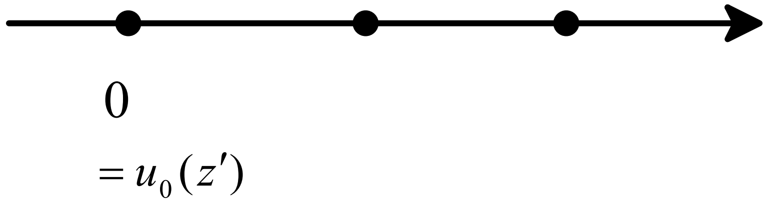
$U_0(z,1) =$

Suppose that $u_0(z') = 0$ and $u_1(z) = u_0(z)$ so that $U_0(z,1) =$

- $u_0(z) < \underbrace{0}_{=u_0(z')}$



- $u_0(z) > \underbrace{0}_{=u_0(z')}$



Ranking sequence of outcomes

EXAMPLE 2.

	Today	*Tomorrow*
date	0	1
Plan A	x	y
Plan B	y	x

Suppose: $u_0(x) = u_1(x) = 4$ $u_0(y) = u_1(y) = 6$ $\delta = 0.8$.

	Today	*Tomorrow*
date	0	1
Plan A		
Plan B		

Extension of the discounted utility:

$U_0(\text{Plan A}) =$

$U_0(\text{Plan B}) =$

EXAMPLE 3.

date	0	1	2
Plan A	x	y	z
Plan B	y	z	x

$U_0(\text{Plan A}) =$

$U_0(\text{Plan B}) =$

Suppose $\begin{cases} \delta = 0.9, \\ u_0(x) = 0,\ u_1(y) = 4,\ u_2(z) = 2 \\ u_0(y) = 3,\ u_1(z) = 1,\ u_2(x) = 1 \end{cases}$, then

date	0	1	2
Plan A			
Plan B			

$U_0(\text{Plan A}) =$

$U_0(\text{Plan B}) =$

Time consistency of preferences

date	0	1	2	3
Plan A	–	x	y	z
Plan B	–	y	z	x

Suppose that you "choose" Plan *B*:

Now when date 1 comes along you re-examine those two plans and are free to change your mind (there was no commitment). Your preferences are **time consistent** if at date 1 you maintain the same ranking that you had at time 0:

Recall

$$U_0(z,t) =$$

Extend this to the preferences at any time *s*:

$$U_s(z,t) =$$ assuming that

$U_s(z,t) =$ assuming that $t \geq s$

	Date 0	Date 1	Date 2	Date 3	Date 4
Plan A	--	--	*x*	*y*	x
Plan B	--	--	y	z	x

$U_0(\text{Plan A}) =$

$U_1(\text{Plan A}) =$

$U_2(\text{Plan A}) =$

And similarly for the utility of Plan B.

Now suppose that at time 0 you prefer Plan A to Plan B:

(**)

Divide both sides of (**) by δ :

Divide both sides of (**) by δ^2 :