

## Arrow's axioms

- **Axiom 1: Unrestricted Domain or Freedom of Expression**

At the individual level, any complete and transitive ranking should be allowed.

- **Axiom 2: Rationality**

Also the social ranking should be complete and transitive

• **Axiom 3: Unanimity or Pareto Principle**

	1's ranking	2's ranking	3's ranking
best	$A$	$C$	$B$
	$B$	$A$	$C$
worst	$C$	$B$	$A$

	1's ranking	2's ranking	3's ranking
best	$A$	$C$	$A, B$
	$B$	$A$	
worst	$C$	$B$	$C$

	1's ranking	2's ranking	3's ranking
best	$A$	$C$	$A$
	$B$	$A$	$C$
worst	$C$	$B$	$B$

- **Axiom 4: Non-dictatorship**

## • Axiom 5: Independence of Irrelevant Alternatives

(1) 

		individual 1	individual 2	
best		$A$	$A, B$	suppose that $\mapsto A \succ B$
		$B$		
worst		$C$	$C$	

		1	2
best		$A$	$A, B, C$
		$B$	
worst		$C$	

		1	2
best		$A$	$C$
		$B$	
worst		$C$	$A, B$

		1	2
best		$C$	$A, B$
		$A$	
worst		$B$	$C$

		1	2
best		$A, C$	$A, B$
worst		$B$	$C$

		1	2
best		$A$	$A, B$
		$C$	
worst		$B$	$C$

		1	2
best		$A$	$A, B$
worst		$B, C$	$C$

		1	2
best		$C$	$A, B, C$
		$A$	
worst		$B$	

		1	2
best		$A, C$	$A, B, C$
worst		$B$	

		1	2
best		$A$	$A, B, C$
		$C$	
worst		$B$	

		1	2
best		$A$	$A, B, C$
worst		$B, C$	

		1	2
best		$C$	$C$
		$A$	
worst		$B$	$A, B$

		1	2
best		$A, C$	$C$
worst		$B$	$A, B$

		1	2
best		$A$	$C$
		$C$	
worst		$B$	$A, B$

		1	2
best		$A$	$C$
worst		$B, C$	$A, B$

If there are only two alternatives the Independence of Irrelevant Alternatives axiom is trivially satisfied.

**Remark 1.** *If there are only two alternatives (and any number of individuals) then the method of majority voting satisfies all of Arrow's axioms.*

## Arrow's Impossibility Theorem

If the number of alternatives is at least three,  
there is no social preference function that satisfies the five axioms.

# Arrow's axioms

**Unrestricted Domain  
or Freedom of Expression**

**Rationality**

**Unanimity or Pareto**

**Non-Dictatorship**

**Independence of Irrelevant Alternatives**

**R**  $\left\{ \begin{array}{l} \text{Completeness} \\ \text{Transitivity} \end{array} \right.$

**U**

**ND**

**IIA**


**Majority Rule with 2 alternatives**

**Plurality Rule with 2 alternatives**

**Majority Rule with more than 2 alternatives**

**Plurality Rule with more than 2 alternatives**

*n* voters

majority =  **if n is even:** number of individuals  $\geq \frac{n}{2} + 1$   
**if n is odd:** number of individuals  $\geq \frac{n+1}{2}$

**Majority rule:** if a majority prefers x to y then society prefers x to y  
if a majority prefers y to x then society prefers y to x  
otherwise society is indifferent between x and y

**Plurality rule:** if the number of individuals who prefer x to y is  
grater than the number of individuals who prefer  
y to x then society prefers x to y

if the number of individuals who prefer y to x is  
grater than the number of individuals who prefer  
x to y then society prefers y to x

otherwise society is indifferent between x and y