Arrow's Impossibility Theorem

If the number of alternatives is at least three,
there is no social preference function that satisfies the five axioms.

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Borda count

- n alternatives, m voters
- each voter submits a *strict* ranking of the alternatives
- for each voter the top alternative receives n points, the second $(n-1)$ points, etc.
- for each alternative we take the sum of each individual score
- alternatives are ranked according to the computed score

	Voter 1	Voter 2	Voter 3	score
best	a	b	c	
	b	a	b	
worst	c	c	a	

Social ranking:

Which of Arrow's axioms does the Borda count satisfy?

1. Unrestricted domain?

2. Rationality?

3. Unanimity?

4. Non-dictatorship?

5. Independence of irrelevant alternatives?

Voter:	1	2	3	4	5	6	7
best	x	a	b	x	a	b	x
	c	x	a	c	x	a	c
	b	c	x	b	c	x	b
worst	a	b	c	a	b	c	a

Social ranking:

Voter:	1	2	3	4	5	6	7
best	c	a	b	c	a	b	c
	b	c	a	b	c	a	b
	a	b	c	a	b	c	a
worst	x	x	x	x	x	x	x

Social ranking:

Kemeny-Young method

For each pair of alternatives, x and y, count:
(1) the number of individuals for whom $x \succ y$; denote it by $\#(x \succ y)$,
(2) the number of individuals for whom $x \sim y$; denote it by $\#(x \sim y)$,
(3) the number of individuals from whom $y \succ x$ denote it by $\#(y \succ x)$.

Next go through all the complete and transitive rankings of X and for each compute a total score by adding up the scores of each pairwise ranking.

Example: $X = \{A, B, C\}$, $S = \{1, 2, 3, 4, 5\}$

	voter 1	voter 2	voter 3	voter 4	voter 5
best	A	B	B	C	B
	B	C	C	A	A
worst	C	A	A	B	C

Ranking	Score
$A \succ B \succ C$	
$A \succ C \succ B$	
$B \succ A \succ C$	
$B \succ C \succ A$	
$C \succ A \succ B$	
$C \succ B \succ A$	

Social ranking:

Which of Arrow's axioms does Kemeny-Young satisfy?

1. Unrestricted domain?

2. Rationality?

3. Unanimity?

requires some proof: see textbook

4. Non-dictatorship?

5. Independence of irrelevant alternatives?

	1	2	3	4	5	6	7
best	A	A	A	B	B	C	C
	B	B	B	C	C	A	A
worst	C	C	C	A	A	B	B

Ranking	Score
$A \succ B \succ C$	
$A \succ C \succ B$	
$B \succ A \succ C$	
$B \succ C \succ A$	
$C \succ A \succ B$	
$C \succ B \succ A$	

Social ranking:

	1	2	3	**4**	**5**	6	7
best	A	A	A	**C**	**C**	C	C
	B	B	B	**B**	**B**	A	A
worst	C	C	C	**A**	**A**	B	B

Ranking	Score
$A \succ B \succ C$	
$A \succ C \succ B$	
$B \succ A \succ C$	
$B \succ C \succ A$	
$C \succ A \succ B$	
$C \succ B \succ A$	

Social ranking: