NAME: ___________________________ University ID: __________________

CIRCLE THE NAME OF YOUR TA: Yingxue Li or Johannes Matschke

If you don’t know the name of your TA, then write your Section Number: ______________

- By writing your name on this exam you certify that you have not violated the University’s Code of Academic Contact (for example, you have not copied from the work of another student and you have not knowingly facilitated cheating by another student).

- If you submit the exam without writing your name and ID, you will get a score of 0 for this exam.

- If you do not stop writing when told so (at the end), a penalty of 10 points will be deducted from your score.
1. [25 points] Consider the following game, where the payoffs are given in the following order (from top to bottom): player 1, player 2, player 3.

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H  L  D  E
1  2  0  2
3  0  4  0
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Note: to answer the following questions you don’t need to write the normal form (you can, if you want to, but you will save a lot of time if you reason on the extensive form without constructing the normal form).

(a) [2 points] Are there values of $y$ for which Player 3 has a strictly dominant strategy (in the corresponding strategic form)? If your answer is Yes, say what values and what strategy, if your answer is No explain why not.

(b) [3 points] Does Player 2 have weakly dominated strategies (in the corresponding strategic form)? (If your answer is Yes, name the strategies and the strategies that dominate them; if your answer is No prove your claim.)
The game reproduced for your convenience:

(c) [4 points] For what values of $y$ does Player 3 have a weakly dominated strategy (in the corresponding strategic form)? Name the strategy.

(d) [10 points] Find all the backward-induction solutions when $x = 1$ and $y = 2$.

(e) [6 points] Assume that $x = 1$ and $y = 1$. Explain why $(B,D,L)$ is not a backward-induction solution. [Write your answer below.]
2. [26 points] Consider the following game, where the payoffs are von Neumann-Morgenstern payoffs:

(a) [5 points] Write the set of pure strategies of Player 2.

(b) [5 points] How many strategies does Player 3 have?

(c) [16 points] Find all the pure-strategy subgame-perfect equilibria. [Write your answer below and on the next page.]
The game reproduced for your convenience:
3. [12 points] Consider a simultaneous two-player second-price auction concerning a single, indivisible good. The game-frame is as follows: $S_1 = S_2 = \{3, 4, 5, 6, 7\}$ (these are the possible bids), the set of outcomes is the set of pairs $(i, p)$ where $i \in \{1, 2\}$ is the winner of the auction and $p \in \{3, 4, 5, 6, 7\}$ is the price that the winner has to pay, and the outcome function is as follows ($b_i$ denotes the bid of Player $i$, $i = 1, 2$):

$$f(b_1, b_2) = \begin{cases} (1, b_2) & \text{if } b_1 \geq b_2 \\ (2, b_1) & \text{if } b_1 < b_2 \end{cases}. $$

The value of the object to Player 1 is $v_1 = 4$ and the value of the object to Player 2 is $v_2 = 6$. We consider preferences that can be described as “selfish and benevolent”. We state them in terms of Player 1, but the same definitions apply to Player 2. What is written in the box below captures the “selfish” component (recall that $x \succ_1 y$ means that Player 1 prefers outcome $x$ to outcome $y$):

| for every $p < v_1$ and for every $p', (1, p) \succ_1 (2, p')$; |  
| for every $p$ and $p'$, $(1, p) \succ_1 (1, p')$ if and only if $p < p'$. |

What is written in the box below captures the “benevolent” component (recall that $x \sim_1 y$ means that Player 1 is indifferent between outcome $x$ and outcome $y$):

| for every $p$ and $p'$, $(2, p) \succ_1 (2, p')$ if and only if $p < p'$; |
| $(2, 7) \sim_1 (1, v_1)$; and everything that follows from the above by transitivity. |

Suppose that it is common knowledge that both players are selfish and benevolent. Find all the pure-strategy Nash equilibria. [Write your answer below.]
4. [37 points] The prison Warden lines up three prisoners in his office: Arthur is the one closest to the Warden’s desk, Bill is behind Arthur and Cliff is behind Bill. He tells them “I am going to blindfold you and then put a hat on each of you; I only have white hats and black hats (lots of both). Then I will remove the blindfolds, so that each of you can see the hats on top of those, and only of those, who are in front of you (thus, for example Cliff can see the color of Bill’s hat as well as the color of Arthur’s hat; nobody can see his own hat). I am also going to give a little hint: at least one hat will be white.” He then proceeds to do that and continues “the first of you who correctly claims to know the color of his own hat and can give a good argument as to why he makes that claim, will be set free. Somebody who makes a wrong claim will be assigned to latrine duty for the rest of his stay here.” There is a long silence and then one of the prisoners says “Sir, I know the color of my hat!”

(a) [4 points] Who is it? What color is his hat? How does he know? [Write your answer below.]

(b) [12 points] Use states and partitions to represent this situation. [Use the space below and on the next page for your answer.]
(c) Let $E$ be the event that represents the proposition “**exactly two hats are white**”.

(c.1) [3 points] Express the event $E$ in terms of states. [Write your answer below.]

$$E = \text{________________________}$$

(c.2) [9 points] Find the following events: $K_A E$ (Arthur knows $E$), $K_B E$ (Bill knows $E$), $K_C E$ (Cliff knows $E$).

$$K_A E = \text{________________________} , \ K_B E = \text{________________________} , \ K_C E = \text{________________________}$$

(d) [9 points] Let $F$ be the event that represents the proposition “**exactly two hats are black**”.

Find the following events: $K_A F$ (Arthur knows $F$), $K_B F$ (Bill knows $F$), $K_C F$ (Cliff knows $F$).

$$K_A F = \text{________________________} , \ K_B F = \text{________________________} , \ K_C F = \text{________________________}$$