1. (a) No, because if player 1 plays A the payoff of player 3 is the same (namely 2) no matter what strategy player 3 chooses.
(b) Yes \((F,D)\) is weakly dominated by \((G,D)\) and \((F,E)\) is weakly dominated by \((G,E)\).
(c) For \(y \neq 2\) (and only those values of \(y\)): if \(y > 2\) then \(H\) weakly dominates \(L\) and if \(y < 2\) then \(L\) weakly dominates \(H\).
(d) One solution is \((A, (G,E), H)\) and the other is \((B, (G,D), L)\).
(e) Because it is not a strategy profile.

2. (a) \(S_2 = \{GA, GB, GC, HA, HB, HC\}\).
(b) \(2 \times 3 = 6\).
(c) At his singleton node on the left, Player 3 will choose \(R\). Given this, at her left node Player 2 will choose \(H\). Now consider the subgame that starts at Player 2’s right node. Its strategic form is as follows:

\[
\begin{array}{c|ccc}
\text{Player 3} & D & E & F \\
\hline
\text{Player 2} & A & 0 & 6 & 4 & 3 & 6 & 1 \\
 & B & 1 & 0 & 3 & 2 & 2 & 1 \\
 & C & 0 & 2 & 2 & 6 & 1 & 8 \\
\end{array}
\]

For Player 2, \(C\) is strictly dominated by \(B\). After deleting \(C\), \(F\) becomes strictly dominated by \(E\) for Player 3. Thus the iterative elimination of strictly dominated strategies leads to the following reduced game:

\[
\begin{array}{c|cc}
\text{Player 3} & D & E \\
\hline
\text{Player 2} & A & 0 & 6 & 4 & 3 \\
 & B & 1 & 0 & 3 & 2 \\
\end{array}
\]

This game has no pure-strategy Nash equilibria. To find the mixed-strategy equilibrium, let \(p\) be the probability with which Player 1 plays \(A\) and \(q\) the probability with which Player 2 plays \(D\). Then \(q\) must be the solution to \(4(1-q) = q + 3(1-q)\), which is \(q = \frac{1}{4}\), and \(p\) must be the solution to \(6p = 3p + 2(1-p)\), which is \(p = \frac{2}{3}\). Hence the Nash equilibrium of the subgame is 
\[
\begin{pmatrix}
A & B & C \\
\frac{2}{3} & \frac{3}{5} & 0 \\
\frac{1}{2} & \frac{1}{2} & 0
\end{pmatrix}
\]

The expected payoff of Player 1 in the subgame is thus \(\left(\frac{2}{3} \times \frac{1}{2} \times 1 \right) + \left(\frac{3}{5} \times \frac{1}{2} \times 1 \right) + \left(\frac{2}{3} \times \frac{1}{2} \times 4 \right) + \left(\frac{1}{2} \times \frac{1}{2} \times 4 \right) = \frac{25}{10} = 2.8\). Hence Player 1 is better off playing \(L\) than \(R\). Thus there is only one subgame-perfect equilibrium, which is as follows:

Player 1’s strategy: \(L\), Player 2’s strategy: 
\[
\begin{pmatrix}
A & B & C \\
\frac{2}{3} & \frac{3}{5} & 0 \\
\end{pmatrix}
\]

Player 3’s strategy: 
\[
\begin{pmatrix}
D & E & F \\
\frac{1}{2} & \frac{1}{2} & 0
\end{pmatrix}
\]

3. **Note: this question was taken from the First Midterm.** There are six Nash equilibria: \((3,4), (3,5), (3,6), (3,7), (6,3), (7,3)\).
4. (a) The prisoner is Arthur; he claims – correctly – that his hat is white and he reasons as follows:

- if my (= Arthur’s) hat were black, then Bill would see it and reason as follows
  - I (= Bill) see that Arthur’s hat is black and Cliff also sees this; thus if my (= Bill’s) hat were black, then Cliff would see two black hats and - using the Warden’s hint that at least one hat is white – would immediately claim to know the color of his hat; Cliff’s silence thus tells me that my hat is white and I can thus claim to know that my hat is white.

Since both Cliff and Bill say nothing (there is a long silence), Arthur correctly concludes that his hat is white.

(b) Let $bwb$ mean that Arthur’s hat is black (the first b) and Bill’s hat is white and Cliff’s hat is black. And similarly for the other possibilities. Then the partitions are as follows:

<table>
<thead>
<tr>
<th>ARTHUR:</th>
<th>bbw</th>
<th>bwb</th>
<th>bww</th>
<th>wbb</th>
<th>wbw</th>
<th>wwb</th>
<th>www</th>
</tr>
</thead>
<tbody>
<tr>
<td>BILL:</td>
<td>bbw</td>
<td>bwb</td>
<td>bww</td>
<td>wbb</td>
<td>wbw</td>
<td>wwb</td>
<td>www</td>
</tr>
<tr>
<td>CLIFF:</td>
<td>bbw</td>
<td>bwb</td>
<td>bww</td>
<td>wbb</td>
<td>wbw</td>
<td>wwb</td>
<td>www</td>
</tr>
</tbody>
</table>

(c) (c.1) $E = \{bww, wbw, wwb\}$ . (c.2) $K_A E = \emptyset$, $K_B E = \emptyset$, $K_C E = \emptyset$

(d) $F = \{bbw, wbb, bwb\}$ . $K_A F = K_B F = \emptyset$, $K_C F = \{bbw\}$. 