1. (a) The prisoner is Dan; he claims – correctly – that his hat is yellow and he reasons as follows:

- if my (=Dan’s) hat were red, then Bill would see it and reason as follows
- I (= Ed) see that Dan’s hat is red and Frank also sees this; thus if my (=Ed’s) hat were red, then Frank would see two red hats and - using the Warden’s hint that at least one hat is yellow – would immediately claim to know the color of his hat; Ed’s silence thus tells me that my hat is yellow and I can thus claim to know that my hat is yellow.

Since both Ed and Frank say nothing (there is a long silence), Dan correctly concludes that his hat is yellow.

(b) Let \( ryr \) mean that Dan’s hat is red (the first \( r \)) and Ed’s hat is yellow and Frank’s hat is red. And similarly for the other possibilities. Then the partitions are as follows:

<table>
<thead>
<tr>
<th>DAN:</th>
<th>rry</th>
<th>ryr</th>
<th>ryy</th>
<th>yrr</th>
<th>yry</th>
<th>yyr</th>
<th>yyy</th>
</tr>
</thead>
<tbody>
<tr>
<td>ED:</td>
<td>rry</td>
<td>ryr</td>
<td>ryy</td>
<td>yrr</td>
<td>yry</td>
<td>yyr</td>
<td>yyy</td>
</tr>
<tr>
<td>FRANK:</td>
<td>rry</td>
<td>ryr</td>
<td>ryy</td>
<td>yrr</td>
<td>yry</td>
<td>yyr</td>
<td>yyy</td>
</tr>
</tbody>
</table>

(c) (c.1) \( A = \{ ryy, yry, yyr \} \). (c.2) \( K_D A = \emptyset \), \( K_E A = \emptyset \), \( K_F A = \emptyset \)

(d) \( B = \{ rry, yrr, ryr \} \). \( K_D B = K_E B = \emptyset \), \( K_F B = \{ rry \} \).

2. **Note: this question was taken from the First Midterm.** There are six Nash equilibria: (8,9), (8,10), (8,11), (8,12), (12,8) and (11,8).

3. (a) \( S_3 = \{ DS, DT, ES, ET, FS, FT \} \). (b) \( 3 \times 2 = 6 \).

(c) At his singleton node on the right, Player 3 will choose \( S \). Given this, at her right node Player 2 will choose \( H \). Now consider the subgame that starts at Player 2’s left node. Its strategic form is as follows:
For Player 2, A is strictly dominated by B. After deleting A, F becomes strictly dominated by D for Player 3. Thus the iterative elimination of strictly dominated strategies leads to the following reduced game:

This game has no pure-strategy Nash equilibria. To find the mixed-strategy equilibrium, let $p$ be the probability with which Player 1 plays B and $q$ the probability with which Player 2 plays D. Then $q$ must be the solution to $3q + (1 - q) = 4q$, which is $q = \frac{1}{4}$, and $p$ must be the solution to $2p + 3(1 - p) = 6(1 - p)$, which is $p = \frac{3}{5}$. Hence the Nash equilibrium of the subgame is

$$
\begin{pmatrix}
    A & B & C & D & E & F \\
    0 & \frac{1}{5} & \frac{2}{5} & \frac{1}{2} & \frac{1}{2} & 0 \\
\end{pmatrix}.
$$

The expected payoff of Player 1 in the subgame is thus

$$
\left(\frac{3}{5} \times \frac{1}{5} \times 4\right) + \left(\frac{1}{5} \times \frac{1}{4} \times 4\right) + \left(\frac{2}{5} \times \frac{1}{4} \times 1\right) + \left(\frac{2}{5} \times \frac{1}{4} \times 1\right) = \frac{28}{10} = 2.8.
$$

Hence Player 1 is better off playing L rather than R. Thus there is only one subgame-perfect equilibrium, which is as follows: Player 1’s strategy: L, Player 2’s strategy:

$$
\begin{pmatrix}
    A & B & C \\
    0 & \frac{1}{5} & \frac{2}{5} \\
\end{pmatrix},
$$

Player 3’s strategy:

$$
\begin{pmatrix}
    D & E & F \\
    \frac{1}{2} & \frac{1}{2} & 0 \\
\end{pmatrix}.
$$

4. (a) No, because if player 1 plays C the payoff of player 3 is the same (namely 8) no matter what strategy player 3 chooses.

(b) Yes $(G,D)$ is weakly dominated by $(F,D)$ and $(G,E)$ is weakly dominated by $(F,E)$.

(c) For $y \neq 2$ (and only those values of $y$): if $y < 2$ then $H$ weakly dominates $L$ and if $y > 2$ then $L$ weakly dominates $H$.

(d) One solution is $(C, (F,E), L)$ and the other is $(A, (F,D), H)$.

(e) Because it is not a strategy profile.