Pete and his three daughters Elise, Andrea and Theresa are in the same room. Pete gives a sealed envelope to Elise and tells her (in a loud voice, so that everybody can hear) “in this envelope I put a sum of money; I won’t tell you how much I put in it, but you should know that it was either $8 or $12 or $16. Now, my dear Elise, go to your room by yourself and open it. Divide the money into two equal amounts, put the two sums in two different envelopes, seal them and give one to Andrea and one to Theresa”. Then he leaves the room. Elise turns to her sisters and tells them: “I never told you this, but I like one of you better than the other. So what I will do is this: after dividing the sum into two equal parts, I will take $1 from one envelope and put it in the other envelope. I will then give the envelope with more money to my favorite sister and the envelope with the smaller amount to the other sister”. Later on Andrea and Theresa get each a sealed envelope from Elise, go to their respective rooms and privately open their envelopes. As expected, they find an odd number of dollars. Neither Andrea nor Theresa suspect that Elise kept some money for herself. In fact, it is common knowledge between them that Elise simply rearranged the money – moving $1 from one envelope to the other – without keeping any money for herself.

(a) Represent the state of knowledge between Andrea and Theresa using a set of states and information partitions.

(b) Let E be the event that Elise favored Andrea. Find the events $K_A \cap E$ (the event that Andrea knows it), $K_T \cap E$ (the event that Theresa knows it), $K_A \cap K_T \cap E$ and $K_T \cap K_A \cap E$.

(c) [Answer this question only if the notion of common knowledge has already been covered in class.] (c.1) Find the common knowledge partition. (c.2) At what states is it common knowledge between Andrea and Theresa that Elise’s favorite sister is Andrea?

(d) The night before Pete was looking through his digital crystal ball and saw what Elise was planning to do. However the Microsoft Predictor™ software was not working properly (the screen kept freezing) and he could not tell whether Elise was trying to favor Andrea or Theresa. He knew he couldn’t stop Elise and wondered what he could do to protect the mistreated sister (whether it be Andrea or Theresa), by making sure that she would not know that she had been mistreated. So he asked the digital mirror on the wall “mirror, mirror on the wall, how much money should I put in the envelope?” The mirror did not answer because “envelope” does not rhyme with “wall”. So, you should answer (riddles are not acceptable: you need to fully explain your answer).