NAME: _______________________________ University ID: _____________________

CIRCLE THE NAME OF YOUR TA: Kepler Illich or Marina Lovchikova or William Volckmann

If you don’t know the name of your TA, then write your Section Number: ________________

• By writing your name on this exam you certify that you have not violated the University’s Code of Academic Contact (for example, you have not copied from the work of another student and you have not knowingly facilitated cheating by another student).

• If you don’t explain (= show your work for) your answers you will get no credit.

• If you submit the exam without writing your name and ID, you will get a score of 0 for this exam.

• If you do not stop writing when told so (at the end), a penalty of 10 points will be deducted from your score.
1. [40 points] Consider the following game:

(a) [6 points] How many proper subgames are there?

(b) (b.1) [4 points] Write one mixed strategy of Player 1.

(b.2) [4 points] Write one behavioral strategy of Player 1.
(c) [26 points] Find the subgame-perfect equilibrium. [Hint: it is not in pure strategies.] [You can use this page and the next.]
The game reproduced for your convenience:

The top number is player 1's von Neumann - Morgenstern payoff
the bottom number is player 2's von Neumann - Morgenstern payoff
2. [40 points] The set of states is \{a,b,c,d,e,f,g,h\}. Recall that, if \( F \) is an event (that is, a set of states), then \( \neg F \) (read “not \( F \)”) denotes the complement of \( F \), that is, the set of states that are not in \( F \) (for example, if \( F = \{a,d,e,f,h\} \) then \( \neg F = \{b,c,g\} \)). There are three individuals with the following information partitions:

\[
\begin{align*}
\text{1:} & \quad a \quad b \quad c \quad d \quad e \quad f \quad g \quad h \\
\text{2:} & \quad a \quad b \quad c \quad d \quad e \quad f \quad g \quad h \\
\text{3:} & \quad a \quad b \quad c \quad d \quad e \quad f \quad g \quad h
\end{align*}
\]

Let \( E = \{a,b,c,f,h\} \). Find the following events:

(a) [5 points] \( K_1E \) (1 knows \( E \)).

(b) [5 points] \( K_2E \) (2 knows \( E \)).

(c) [10 points] \( \neg K_3 K_2 \neg K_1 E \) (it is not the case that 3 knows that 2 knows that 1 does not know \( E \)).

(d) [10 points] \( CK_12E \) (\( E \) is common knowledge between 1 and 2).

(e) [10 points] \( \neg K_3 CK_12E \) (it is not the case that 3 knows that \( E \) is commonly known between 1 and 2).
3. [20 points] There are 7 possible states: \( a, b, c, d, e, f \) and \( g \). Initially you have the following beliefs:

\[
\begin{array}{ccccccc}
  a & b & c & d & e & f & g \\
  \frac{1}{20} & \frac{1}{10} & \frac{3}{20} & \frac{1}{10} & \frac{3}{20} & \frac{1}{4} & \frac{1}{5} \\
\end{array}
\]

(a) [4 points] What probability do you assign to event \( E = \{a, c, d, g\} \)?

(b) Suppose that you now receive information represented by the event \( F = \{b, c, e, f, g\} \).

(b.1) [8 points] What are your revised beliefs in light of this information? Answer by filling in the following table:

\[
\begin{array}{ccccccc}
  a & b & c & d & e & f & g \\
  \text{probability} & & & & & & \\
\end{array}
\]

(b.2) [8 points] What probability do you assign to event \( E = \{a, c, d, g\} \) in light of the new information?