1.  
(a) There are two proper subgames: one starting at the node of player 2 on the left-hand side and the other starting at the node of player 2 on the right-hand side.

(b) (b.1) Any probability distribution over the set \{LA, LB, LC, RA, RB, RC\}.
(b.2) Any pair of probability distributions, one over the set \{L,R\} and one over the set \{A,B,C\}.

(c) We first solve the subgames. The subgame on the right is a trivial game, whose equilibrium is for player 2 to choose H. The normal-form of the subgame on the left is as follows:

<table>
<thead>
<tr>
<th></th>
<th>(q) D</th>
<th>(0) E</th>
<th>(1-q) F</th>
</tr>
</thead>
<tbody>
<tr>
<td>p A</td>
<td>2, 1</td>
<td>2, 0</td>
<td>3, 2</td>
</tr>
<tr>
<td>(0) B</td>
<td>1, 0</td>
<td>1, 2</td>
<td>2, 4</td>
</tr>
<tr>
<td>(1-p)C</td>
<td>1, 4</td>
<td>0, 1</td>
<td>4, 2</td>
</tr>
</tbody>
</table>

This game does not have any pure-strategy Nash equilibria. Thus we need to find a mixed-strategy equilibrium. Note that a strictly dominated strategy can never be played with positive probability at a mixed-strategy Nash equilibrium. B is strictly dominated for Player 1 (by A), thus Player 1 must assign zero probability to B. Similarly, E is strictly dominated for Player 2 (by F), thus Player 2 must assign zero probability to E. Thus a mixed-strategy equilibrium must be of the form \( \begin{pmatrix} A & B & C \\ p & 0 & 1-p \end{pmatrix} \begin{pmatrix} D & E & F \\ q & 0 & 1-q \end{pmatrix} \). To find the equilibrium we have to solve: \( 2q + 3(1-q) = q + 4(1-q) \) for \( q \) and \( p + 4(1-p) = 2p + 2(1-p) \) for \( p \). The solution is: \( p = \frac{2}{3} \), and \( q = \frac{1}{2} \). Thus the mixed-strategy Nash equilibrium of the subgame is given by:

\[
\begin{pmatrix} A & B & C \\ \frac{2}{3} & 0 & \frac{1}{2} \end{pmatrix} \begin{pmatrix} D & E & F \\ \frac{1}{2} & 0 & \frac{1}{2} \end{pmatrix}
\]

Player 1’s expected payoff is \( \frac{2}{3} \times \frac{1}{2} \times 2 + \frac{2}{3} \times \frac{1}{2} \times 3 + \frac{1}{3} \times \frac{1}{2} \times 1 + \frac{1}{3} \times \frac{1}{2} \times 4 = \frac{15}{6} = \frac{5}{2} = 2.5 \).

Player 2’s expected payoff is \( \frac{2}{3} \times \frac{1}{2} \times 1 + \frac{2}{3} \times \frac{1}{2} \times 2 + \frac{1}{3} \times \frac{1}{2} \times 4 + \frac{1}{3} \times \frac{1}{2} \times 2 = \frac{12}{6} = 2 \).

Thus the player 1 will want to choose L. Hence the subgame-perfect equilibrium of the entire game, given in terms of behavioral strategies is

\[
\begin{pmatrix} L & R \\ 1 & 0 \end{pmatrix} \begin{pmatrix} A & B & C \\ \frac{2}{3} & 0 & \frac{1}{3} \end{pmatrix} \begin{pmatrix} D & E & F \\ \frac{1}{2} & 0 & \frac{1}{2} \end{pmatrix} \begin{pmatrix} G & H \\ 0 & 1 \end{pmatrix}
\]
with expected payoffs of 2.5 for player 1 and 2 for player 2.

In terms of mixed strategies the equilibrium is given by

\[
\begin{pmatrix}
LA & LB & LC & RA & RB & RC & GD & GE & GF & HD & HE & HF \\
\frac{2}{3} & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2}
\end{pmatrix}
\]

2. Note: This question was based on an example given in class on February 27.

(a) \( K_1E = \{a, b, c, f\} \)

(b) \( K_2E = \{a, b, c, f, h\} = E \)

(c) Since \( K_1E = \{a, b, c, f\} \), \( \neg K_1E = \{d, e, g, h\} \); thus

\( K_2 \neg K_1 = K_2 \{d, e, g, h\} = \{d, e, g\} \) and \( K_3K_2 \neg K_1 = K_3 \{d, e, g\} = \{g\} \). Hence

\( \neg K_3K_2 \neg K_1E = \{a, b, c, d, e, f, h\} \)

(d) The common knowledge partition between individuals 1 and 2 is

\( \{\{a, b, c\}, \{d, e\}, \{a, e, f, g, h\}\} \). Thus \( CK_{12}E = \{a, b, c\} \).

(e) Since \( CK_{12}E = \{a, b, c\}, \neg K_3CK_{12}E = \{b, c\} \) so that \( \neg K_3CK_{12}E = \{a, d, e, f, g, h\} \).

3. First convert all the probabilities in terms of a common denominator:

<table>
<thead>
<tr>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
<th>e</th>
<th>f</th>
<th>g</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\frac{1}{20})</td>
<td>(\frac{3}{20})</td>
<td>(\frac{1}{20})</td>
<td>(\frac{3}{20})</td>
<td>(\frac{3}{20})</td>
<td>(\frac{5}{20})</td>
<td>(\frac{4}{20})</td>
</tr>
</tbody>
</table>

(a) \( P(E) = \frac{1}{20} + \frac{3}{20} + \frac{3}{20} + \frac{4}{20} = \frac{1}{2} = 50\% \)

(b) (b.1) The revised beliefs are as follows:

<table>
<thead>
<tr>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
<th>e</th>
<th>f</th>
<th>g</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0)</td>
<td>(\frac{2}{17})</td>
<td>(\frac{3}{17})</td>
<td>(0)</td>
<td>(\frac{3}{17})</td>
<td>(\frac{5}{17})</td>
<td>(\frac{4}{17})</td>
</tr>
</tbody>
</table>

(b.2) The revised probability of \( E \) is \( \frac{2}{17} + \frac{3}{17} = \frac{5}{17} = 41.18\% \).