1. (a) One (starting at Player 2’s node after choice C of Player 1).
(b) Player 2’s pure strategies are: DF, DG, EF and EG.
(c) $\begin{pmatrix} DF & DG & EF & EG \\ p & q & r & 1-p-q-r \end{pmatrix}$ for any three numbers $p, q$ and $r$ between 0 and 1 (with at least one strictly between 0 and 1).
(d) $\begin{pmatrix} D & E & F & G \\ p & 1-p & q & 1-q \end{pmatrix}$ for any two numbers $p$ and $q$ between 0 and 1 (with at least one strictly between 0 and 1).
(e) There is only one proper subgame, starting from the node of player 2 after choice C of player 1. The normal form of that game is

```
  F  2  2  1  0
Player 3
  G  3  5  0  6
```

This game does not have pure-strategy Nash equilibria. To find the mixed-strategy Nash equilibrium, let $p$ be the probability of $F$ and $q$ the probability of $L$. Then $q$ must satisfy the equation $2q + 1(1-q) = 3q + 0(1-q)$.

Thus $q = \frac{1}{3}$. Similarly, $p$ must satisfy the equation $2p + 5(1-p) = 0(p) + 6(1-p)$ whose solution is $p = \frac{2}{3}$. Thus the Nash equilibrium of the subgame is $\begin{pmatrix} F & G & L & M \\ 1/3 & 2/3 & 1/3 & 1/3 \end{pmatrix}$ with corresponding expected payoffs of 1.5 for player 2 and 4 for player 3. The expected payoff of player 1 in the subgame is $2(\frac{2}{3}) + 2(\frac{1}{3}) + 4(\frac{1}{3}) + 0(\frac{1}{3}) = 2$. Thus the game can be simplified as shown below:

```
        1.5
      /     \
     C      D
      \
        1.5
    4
  2
```

In this game, for player 1 both A and B are strictly dominated by C. Thus at any Nash equilibrium player 1 must play C with probability 1. The pure strategy C together with any mixture of D and E is a Nash equilibrium. Hence there is an infinite number of subgame-perfect equilibria of this game as follows: for every $0 \leq r \leq 1$, $\begin{pmatrix} A & B & C & D & E & F & G & L & M \\ 0 & 0 & 1 & r & 1-r & 1/3 & 2/3 & 1/2 & 1/2 \end{pmatrix}$ is a subgame-perfect equilibrium. The payoff are the same for all the equilibria, namely (2,1.5,4).

```
2. Note: this was Exercise 8 in Chapter 7 of the book. (a) We can represent a state as a triple $(x, y, z)$ where $x$ is the position of the switch in room 1 (Up or Down), $y$ is the position of the switch in room 2 and $z$ is the light which is on (Green or Red):

\[
\begin{array}{ccc}
U & U & D \\
U & D & U \\
G & R & R \\
G & R & G
\end{array}
\]

(b)

Ann

\[
\begin{array}{ccc}
U & U & D \\
U & D & D \\
G & R & R \\
G & R & G
\end{array}
\]

Bob

\[
\begin{array}{ccc}
U & U & D \\
U & D & D \\
G & R & R \\
G & R & G
\end{array}
\]

Carla

\[
\begin{array}{ccc}
U & U & D \\
U & D & D \\
G & R & R \\
G & R & G
\end{array}
\]

(c) (c.1) $G = \left\{ U \bigg| D \right\}$ (c.2)-(c.4) $K_A G = \emptyset$, $K_B G = \emptyset$, $K_C G = \left\{ U \bigg| D \right\}$.

(d) (d.1) $L$ is the set of all states. (d.2) $\neg K_A L = \emptyset$ (d.3) $K_B K_C L = L$ (d.4) $K_C \neg K_A L = \emptyset$ (d.5) $CKL = L$.