1. Note: this question was based on an example given in class on February 27.
(a) $K_1E = \{a, b, c, d, g\}$
(b) $K_2E = \{a, b, c, d, e\}$
(c) Since $K_1E = \{a, b, c, d, g\}$, $\neg K_1E = \{e, f, h\}$; thus $K_2\neg K_1E = K_2\{e, f, h\} = \{e, h\}$ and $K_3K_2\neg K_1E = K_3\{e, h\} = \{e\}$. Hence $\neg K_1K_2\neg K_1E = \{a, b, c, d, f, g, h\}$
(d) The common knowledge partition between individuals 1 and 2 is $\{\{a, b, c, d\}, \{e, f, g\}, \{h\}\}$. Thus $CK_{12}E = \{a, b, c, d\}$.
(e) Since $CK_{12}E = \{a, b, c, d\}$, $K_3CK_{12}E = \{a, b, c, d\}$ so that $\neg K_3CK_{12}E = \{e, f, g, h\}$.

2. (a)
   (a) Here are two proper subgames: one starting at the node of player 2 on the left-hand side and the other starting at the node of player 2 on the right-hand side.
(b) (b.1) Any probability distribution over the set \{GD, GE, GF, HD, HE, HF\}.
(b.2) Any pair of probability distributions, one over the set \{G,H\} and the other over the set \{D,E,F\}.
(c) We first solve the subgames. The subgame on the right is a trivial game, whose equilibrium is for player 2 to choose H. The normal-form of the subgame on the left is as follows:

<table>
<thead>
<tr>
<th></th>
<th>(q) D</th>
<th>(0) E</th>
<th>(1−q) F</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0) A</td>
<td>1, 0</td>
<td>1, 2</td>
<td>2, 4</td>
</tr>
<tr>
<td>(p) B</td>
<td>2, 1</td>
<td>2, 0</td>
<td>3, 2</td>
</tr>
<tr>
<td>(1−p)C</td>
<td>1, 4</td>
<td>0, 1</td>
<td>4, 2</td>
</tr>
</tbody>
</table>

This game does not have any pure-strategy Nash equilibria. Thus we need to find a mixed-strategy equilibrium. Note that a strictly dominated strategy can never be played with positive probability at a mixed-strategy Nash equilibrium. A is strictly dominated for Player 1 (by B), thus Player 1 must assign zero probability to A. Similarly, E is strictly dominated for Player 2 (by F), thus Player 2 must assign zero probability to E. Thus a
mixed-strategy equilibrium must be of the form \[ \begin{bmatrix} A & B & C \\ p & 1-p & 1-p \\ q & 0 & 1-q \end{bmatrix} \]. To find the equilibrium we have to solve:

\[ 2q + 3(1-q) = q + 4(1-q) \] for \( q \) and \( p + 4(1-p) = 2p + 2(1-p) \) for \( p \). The solution is: \( p = \frac{2}{3} \), and \( q = \frac{1}{2} \). Thus the mixed-strategy Nash equilibrium of the subgame is given by:

\[ \begin{bmatrix} A & B & C & D & E & F \\ 0 & \frac{2}{3} & \frac{1}{3} & \frac{1}{2} & 0 & \frac{1}{2} \end{bmatrix} \]

Player 1’s expected payoff is

\[ \frac{2}{3} \times 1 + \frac{1}{3} \times 2 + \frac{3}{2} \times 3 + \frac{1}{2} \times 1 + \frac{1}{3} \times 2 = \frac{15}{6} = \frac{5}{2} = 2.5 \]

Player 2’s expected payoff is

\[ \frac{2}{3} \times 1 + \frac{1}{3} \times 2 + \frac{3}{2} \times 1 + \frac{3}{2} \times 4 + \frac{1}{3} \times 2 = \frac{12}{6} = 2 \].

Thus the player 1 will want to choose R. Hence the subgame-perfect equilibrium of the entire game, given in terms of behavioral strategies is

\[ \begin{bmatrix} L & R & A & B & C & D & E & F \\ 0 & 1 & 0 & \frac{2}{3} & \frac{1}{3} & \frac{1}{2} & 0 & \frac{1}{2} \end{bmatrix} \]

with expected payoffs of 2.5 for player 1 and 2 for player 2.

In terms of mixed strategies the equilibrium is given by

\[
\begin{bmatrix}
L & A & B & C & D & E & F & G & H \\
0 & 0 & 0 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} \\
\end{bmatrix}
\]

3. First convert all the probabilities in terms of a common denominator:

\[
\begin{bmatrix}
a & b & c & d & e & f & g \\
\frac{1}{20} & \frac{2}{20} & \frac{3}{20} & \frac{2}{20} & \frac{3}{20} & \frac{5}{20} & \frac{4}{20} \\
\end{bmatrix}
\]

(a) \( E = \{b, c, f, g\} \), \( P(E) = \frac{2}{20} + \frac{3}{20} + \frac{4}{20} + \frac{4}{20} = \frac{14}{20} = 70\% \).

(b) \( F = \{a, c, d, f, g\} \). The revised beliefs are as follows:

\[
\begin{bmatrix}
a & b & c & d & e & f & g \\
\frac{1}{15} & 0 & \frac{3}{15} & \frac{2}{15} & 0 & \frac{5}{15} & \frac{4}{15} \\
\end{bmatrix}
\]

(b.1) The revised probability of \( E \) is \( \frac{3}{15} + \frac{5}{15} + \frac{4}{15} = \frac{12}{15} = 80\% \).

(b.2) The revised probability of \( E \) is \( \frac{3}{15} + \frac{5}{15} + \frac{4}{15} = \frac{12}{15} = 80\% \).