1. **Note: this was Exercise 8 in Chapter 7 of the book.** (a) We can represent a state as a triple \( x,y,z \) where \( x \) is the position of the switch in room 1 (Up or Down), \( y \) is the position of the switch in room 2 and \( z \) is the light which is on (Green or Red):

\[
\begin{array}{ccc}
U & D & D \\
U & D & U \\
G & R & R \\
\end{array}
\]

(b)

<table>
<thead>
<tr>
<th></th>
<th>Ann</th>
<th>Bob</th>
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<tbody>
<tr>
<td>U</td>
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<td>U</td>
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<td>R</td>
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(c) (c.1) \( E \) is the set of all states.  
(c.2) \( \neg K_x E = \emptyset \)  
(c.3) \( K_y K_z E = E \)

(c.4) \( K_c \neg K_x E = \emptyset \)  
(c.5) \( CKE = E \).

(d) (d.1) \( R = \begin{pmatrix} U & D \\ D & U \\ R & R \end{pmatrix} \)  
(d.2)-(d.4) \( K_x R = \emptyset, \ K_y R = \emptyset, \ K_z R = \begin{pmatrix} U & D \\ D & U \\ R & R \end{pmatrix} = R. \)
2. (a) One (starting at Player 2’s node after choice A of Player 1).

(b) Player 2’s pure strategies are: DH, DL, EH and EL.

(c) \[
\begin{pmatrix}
DH & DL & EH & EL \\
p & q & r & 1-p-q-r
\end{pmatrix}
\] for any three numbers \(p, q\) and \(r\) between 0 and 1 (with at least one strictly between 0 and 1).

(d) \[
\begin{pmatrix}
D & E & H & L \\
p & 1-p & q & 1-q
\end{pmatrix}
\] for any two numbers \(p\) and \(q\) between 0 and 1 (with at least one strictly between 0 and 1).

(e) There is only one proper subgame, starting from the node of Player 2 after choice A of Layer 1. The normal form of that game is

\[
\begin{array}{c|cc}
& F & G \\
D & 6 & 2 \\
E & 9 & 5
\end{array}
\]

This game does not have pure-strategy Nash equilibria. To find the mixed-strategy Nash equilibrium, let \(p\) be the probability of D and \(q\) the probability of F. Then \(q\) must satisfy the equation \(6q + 3(1-q) = 9q + 0(1-q)\). Thus \(q = \frac{1}{3}\). Similarly, \(p\) must satisfy the equation \(2p + 5(1-p) = 0(p) + 6(1-p)\) whose solution is \(p = \frac{1}{3}\). Thus the Nash equilibrium of the subgame is \(D E H L \begin{pmatrix} 
F & G \\
\frac{1}{3} & \frac{2}{3} & \frac{1}{2} & \frac{1}{2}
\end{pmatrix}\) with corresponding expected payoffs of 4.5 for Player 2 and 4 for Player 3. The expected payoff of Player 1 in the subgame is \(4\left(\frac{1}{3}\right) + 4\left(\frac{1}{3}\right) + 8\left(\frac{1}{3}\right) + 0\left(\frac{1}{3}\right) = 4\). Thus the game can be simplified as follows:

\[
\begin{array}{c|cc}
& F & G \\
D & 6 & 2 \\
E & 9 & 5
\end{array}
\]

In this game, for Player 1 both C and D are strictly dominated by A. Thus at any Nash equilibrium Player 1 must play A with probability 1. The pure strategy A together with any mixture of H and L is a Nash equilibrium. Hence there is an infinite number of subgame-perfect equilibria of this game as follows: for every \(0 \leq r \leq 1\),

\[
\begin{pmatrix}
A & B & C & D & E & H & L & F & G \\
1 & 0 & 0 & 1 & 2 & r & 1-r & \frac{1}{2} & \frac{1}{2}
\end{pmatrix}
\] is a subgame-perfect equilibrium. The payoff are the same for all the equilibria, namely (4,4.5,4).