1. 
(a) 10 is not strictly dominated for Player 1 (if Player 2 writes 90, then writing 10 gives Player 1 a payoff of $10, while every other strategy gives him $0).

(b) Player 1 does not have strictly dominated strategies.

(c) Yes, 100 is weakly dominated (for example by 10). In fact, writing 100 gives Player 1 $0 in every possible case, while writing, say 10, would give him 10 in some cases and 0 in others.

(d) The only weakly dominated strategy is 100.

(e) The Nash equilibria are all and only the pairs (x,y) such that x + y = 100, as well as the pair (100,100)

2. The assumption is that each player only cares about how much money he/she gets and prefers more money to less. Thus we can take for each player as utility function the identity function: \( U(\$x) = x \) (that is, in the given table, remove the dollar signs and interpret those numbers as utilities). (a) M is a dominant strategy for player 1. (b) Player 2 does not have a dominant strategy. (c) Hence there is no dominant strategy equilibrium. (d) The iterated dominant strategy equilibrium is (M,C).

3. 
(a) \((1,72) \succ_1 (2,35) \sim_1 (2,60)\).

(b) \((2,46) \succ_1 (2,57) \succ_1 (1,73)\).

(c) \((1,18) \succ_1 (2,50) \succ_1 (2,39)\).

(d) This is the standard case where Vickrey’s theorem applies: \( b_1 = v_1 \) is a weakly dominant strategy.

(e) Recall that \( p_1 < v_i < p_m \). Bidding \( v_i \) is not a dominant strategy: if Player 2 bids \( p_m \) then the outcome is \((2,v_i)\) and Player 1 would prefer bidding \( p_1 \), since – by benevolence – \((2,p_1) \succ_1 (2,v_i)\).
Recall that $p_i < v_i < p_m$. We need to distinguish two cases.

**CASE 1**: $v_i < p_{m-1}$. Then if Player 2 bids $p_m$, the outcome of $(v_1, p_m)$ is $(2, v_1)$ and if Player 1 increased his bid to $p_{m-1}$ then the outcome would be $(2, p_{m-1})$ which Player 1 prefers to $(2, v_1)$, since $p_{m-1} > v_i$. Thus bidding $v_i$ is not a dominant strategy for Player 1.

**CASE 2**: $v_i = p_{m-1}$. Then if Player 2 bids $p_m$ the outcome of $(v_1, p_m)$ is $(2, p_m)$ and if Player 1 increased his bid to $p_m$ then the outcome would be $(1, p_m)$ which he considers just as good as $(2, p_m)$, which, in turn, is better than $(2, p_{m-1})$.

The assumption is that it is common knowledge that both players are selfish and **uncaring** and $B = \{1, 2, 3 = v_1, 4, 5 = v_2\}$. Since bidding one’s own true value is a weakly dominant strategy, $(3, 5)$ is a Nash equilibrium; however it is not the only Nash equilibrium. All of the following are Nash equilibria: $(1, 5), (2, 5), (3, 5), (4, 5), (1, 4), (2, 4), (3, 4), (1, 3), (2, 3), (5, 1), (5, 2)$ and $(5, 3)$. Thus a total of 12 equilibria.

The assumption is that it is common knowledge that both players are selfish and **benevolent** and $B = \{1, 2, 3 = v_1, 4, 5 = v_2\}$.

**(h.1)** $(3, 5)$ is not a Nash equilibrium because the associated outcome is $(2, 3)$; by benevolence, $(2, 1) \succeq (2, 3)$ and Player 1 can induce outcome $(2, 1)$ by reducing her bid from 3 to 1.

**(h.2)** The Nash equilibria where Player 2 wins the auction are $(1, 3), (1, 4)$ and $(1, 5)$.

The assumption is that it is common knowledge that both players are selfish and **spiteful** and $B = \{1, 2, 3 = v_1, 4, 5 = v_2\}$.

**(i.1)** $(3, 5)$ is not a Nash equilibrium because the associated outcome is $(2, 3)$; by spitefulness, $(2, 4) \succ (2, 3)$ and Player 1 can induce outcome $(2, 4)$ by increasing her bid from 3 to 4.

**(i.2)** Nash equilibria where the outcome is that Player 2 wins the auction are $(4, 5)$ and $(3, 4)$. 