PRACTICE FOR FIRST MIDTERM EXAM
Answer all questions. Explain your answers.

1. Two players can make a total profit of $100 if they are not too greedy. The players are in different rooms and cannot communicate. Each player has to write a multiple of 10 not greater than 100 (thus either 10 or 20 or 30 … or 100) on a piece of paper and hand it to the referee. The referee opens the two bids and does the following:
   - if the two amounts written down add up to 100 or less, she gives each player the amount he wrote and the rest (if any) is given to SFWGB (the Special Fund for the Welfare of Giacomo Bonanno) [thus if Player 1 writes 20 and Player 2 writes 60, then Player 1 gets $20 and Player 2 gets $60],
   - if the two amounts written down add up to more than $100, the referee gives nothing to the players and $100 to SFWGB.

Each player only cares about how much money he gets: he does not care about how much money the other player gets (e.g. whether she gets more or less than him) nor does he care about the SFWGB (which is something he should be ashamed of!).

(a) Consider Player 1. Is the strategy of writing 10 strictly dominated by another strategy?
(b) What are Player 1’s strictly dominated strategies?
(c) Consider Player 1. Is the strategy of writing 100 weakly dominated by another strategy?
(d) What are Player 1’s weakly dominated strategies?
(e) What are the Nash equilibria of this game?

2. Consider the following game, where in each cell the first number if the sum of money that Player 1 gets and the second number is the sum of money that Player 2 gets. Both players are selfish and greedy.

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<tbody>
<tr>
<td>T</td>
<td>$10, $18</td>
<td>$7, $20</td>
<td>$1, $18</td>
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<tr>
<td>M</td>
<td>$12, $15</td>
<td>$8, $16</td>
<td>$1, $12</td>
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<tr>
<td>B</td>
<td>$10, $9</td>
<td>$4, $8</td>
<td>$0, $0</td>
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(a) Does player 1 have a weakly dominant strategy?
(b) Does player 2 have a weakly dominant strategy?
(c) Is there a dominant-strategy equilibrium?
(d) Is there an iterated dominant-strategy equilibrium?
3. Consider a simultaneous two-player second-price auction concerning a single indivisible good. The game-frame is as follows: \( S_i = S_2 = B \) where \( B = \{p_1, p_2, \ldots, p_m\} \) is a finite set of positive numbers with \( p_1 < p_2 < \ldots < p_m \), the set of outcomes is the set of pairs \((i, p)\) where \( i \in \{1, 2\} \) is the winner of the auction and \( p \in B \) is the price that the winner has to pay and the outcome function is as follows (\( b_i \) denotes the bid of Player \( i \)): 

\[
f(b_1, b_2) = \begin{cases} 
(1, b_2) & \text{if } b_1 \geq b_2 \\
(2, b_1) & \text{otherwise}
\end{cases}
\]

Let \( v_i \) be the value of the object to Player \( i \) (that is, Player \( i \) views getting the object as equivalent to getting \$ \( v_i \)). We shall consider various kinds of preferences. We state them in terms of Player 1, but the same definitions apply to Player 2. The following apply to all three preferences (this is the “\textit{selfish}” part):

- for every \( p < v_i \) and for every \( p', (1, p) \succ (2, p') \);
- for every \( p \) and \( p' \), \((1, p) \succ (1, p') \) if and only if \( p < p' \).

- Player 1 is \textit{selfish and uncaring} if, in addition, her preferences are as follows:
  - for every \( p \) and \( p' \), \((2, p) \sim (2, p') \);
  - for every \( p \), \((2, p) \sim (1, v_i) \);
  - and everything that follows from the above by transitivity.

- Player 1 is \textit{selfish and benevolent} if, in addition, her preferences are as follows:
  - for every \( p \) and \( p' \), \((2, p) \succ (2, p') \) if and only if \( p < p' \);
  - \((1, v_i) \sim (2, p_i) \);
  - and everything that follows from the above by transitivity.

- Player 1 is \textit{selfish and spiteful} if her preferences are as follows:
  - for every \( p \) and \( p' \), \((2, p) \succ (2, p') \) if and only if \( p > p' \);
  - \((1, v_i) \sim (2, p_m) \);
  - and everything that follows from the above by transitivity.

(a) [5 points] Suppose that Player 1 is \textit{selfish and uncaring} and \( v_i = 80 \). How does she rank the following three outcomes: \((1, 72), (2, 60), (2, 35)\)?

(b) [5 points] Suppose that Player 1 is \textit{selfish and benevolent} and \( v_i = 64 \). How does she rank the following three outcomes: \((1, 73), (2, 57), (2, 46)\)?

(c) [5 points] Suppose that Player 1 is \textit{selfish and spiteful} and \( v_i = 25 \). How does she rank the following three outcomes: \((1, 18), (2, 50), (2, 39)\)?
In parts (d)-(f) assume that $m > 3$, $v_1, v_2 \in B$, $p_i < v_i < p_m$ and $p_i < v_2 < p_m$.

(d) [10 points] Suppose that Player 1 is selfish and \textit{uncaring}. Does she have a weakly or strictly dominant strategy? If your answer is Yes, say what that strategy is and state whether it is weak or strict dominance; if your answer is No prove it.

(e) [10 points] Suppose that Player 1 is selfish and \textit{benevolent}. Is bidding $v_i$ a dominant strategy? Fully explain your answer.

(f) [10 points] Suppose that Player 1 is selfish and \textit{spiteful}. Is bidding $v_i$ a dominant strategy? Fully explain your answer.

(g) [15 points] Suppose that it is common knowledge that both players are selfish and \textit{uncaring}, $B = \{1,2,3,4,5\}, v_1 = 3$ and $v_2 = 5$. Find all the pure-strategy Nash equilibria.

(h) Suppose that it is common knowledge that both players are selfish and \textit{benevolent}, $B = \{1,2,3,4,5\}, v_1 = 3$ and $v_2 = 5$.

(h.1) [5 points] Is (3,5) a Nash equilibrium? Explain your answer.

(h.2) [15 points] Find three Nash equilibria where the outcome is that Player 2 wins the auction.

(i) Suppose that it is common knowledge that both players are selfish and \textit{spiteful}, $B = \{1,2,3,4,5\}, v_1 = 3$ and $v_2 = 5$.

(i.1) [5 points] Is (3,5) a Nash equilibrium? Explain your answer.

(i.2) [15 points] Find two Nash equilibria where the outcome is that Player 2 wins the auction.