1. Amy, Beth and Carla are now in room 1. They are asked to proceed, one at a time, to room 3 through room 2. In room 2 there are two large containers, one with red hats only and the other with white hats only. They have to choose one hat (obviously each of them knows the color of the hat she chooses), put it on their head and then go and sit in the chair in room 3 that has their name on it.

Amy goes first, then Beth then Carla. The chairs are with the back to the door. Thus a person entering room 3 can see whoever is already seated there, but cannot be seen by them. Suppose that Amy chooses a white hat, Beth a red hat and Carla a white one.

(a) Represent the epistemic situation that arises when they are all seated in room 3 (that is, use an information partition model to represent what each individual knows).

(b) Find a smallest event that is common knowledge among them. Give also a verbal interpretation of this event.

(c) Repeat (a) and (b) assuming now that there is a mirror in room 3 that allows Amy to see the hat of Carla (but not that of Beth).

2. Consider the following game. There are two players. Each has to decide whether or not to contribute to public television. If neither of them contributes, then there is no public television and each gets a payoff of zero. If a player decides to contribute, she gives her credit card number to the potential provider of public television. The provider then proceeds as follows: if both players supplied their credit card numbers, each credit card is charged $2 and the broadcasting takes place; if only one player provided her credit card number, then the card is charged $3 and the broadcasting takes place. Public television is a public good, that is, once it is provided everybody can enjoy it, whether or not she contributed to it. Each player derives a benefit from being able to watch public television equivalent to getting $4. The players make their decisions independently of each other (i.e. with no information about what the other player chose).

Write the normal form of this game (there are only two players, namely the potential contributors; the provider of public television is not considered as a player). Give as payoff the net benefit to each player.

Find all the Nash equilibria of this game, including the mixed-strategy ones.