1. (a) For $x < 3$. The strictly dominant strategy is C.

(b) For $x = 3$. The weakly dominant strategy is C.

(c) For no values (if $y \leq 2$ $G$ weakly dominates $H$, however, $G$ does not dominate $F$).

(d) • for any $x$, if $y > 2$ then (C,H) is the only pure-strategy Nash equilibrium.
   • if $x > 3$ and $y < 2$ then (C,G) is the only pure-strategy Nash equilibrium.
   • $x \leq 3$ and $y = 2$, then there are 3 pure-strategy Nash equilibria: (C,F), (C,G), and (C,H).

(e) • $x \leq 3$ and $y < 2$, then (C,F) and (C,G) are the only pure-strategy Nash equilibria.
   • $x > 3$ and $y = 2$, then (C,G) and (C,H) are the only pure-strategy Nash equilibria.

To answer (f) and (g) first note that, for every pair of values of $x$ and $y$, in the first round of elimination strategies A, B and E of Player 1 are among those that get eliminated.
If $y > 2$ then H cannot be deleted in the first round, while if $y \leq 2$ H can be deleted in the first round.
If $x > 3$ then D cannot be deleted in the first round, while if $x \leq 3$ D can be deleted in the first round. Thus:

- If $x > 3$ and $y > 2$ then in the first round delete A, B and E, in the second round delete F and G and in the third round delete D. Left with (C,H).
- If $x \leq 3$ and $y > 2$ then in the first round delete A, B, D and E and in the second round delete F and G. Left with (C,H).
- If $x > 3$ and $y \leq 2$ then in the first round delete A, B, E and H, in the second round delete F and in the third round delete D. Left with (C,G).
- If $x \leq 3$ and $y \leq 2$ then in the first round delete A, B, D, E and H and then no more deletions are possible. Left with (C,F) and (C,G).

Hence:

(f) For $x \leq 3$ and $y \leq 2$.

(g) For $x > 3$ and any value of $y$, or for $y > 2$ and $x \leq 3$.

2. (a) The profit functions are $\pi_1 = q_1[90 - 3(q_1 + q_2)] - 9q_1$ and $\pi_2 = q_2[90 - 3(q_1 + q_2)] - 9q_2$. The Cournot-Nash equilibrium is given by the solution to $\frac{\partial \pi_1}{\partial q_1} = 0$ and $\frac{\partial \pi_2}{\partial q_2} = 0$. The solution is $q_1 = q_2 = 9$ with corresponding profits $\pi_1 = \pi_2 = 243$.

(b) (b.1) Now firm 1 chooses $q_1$ to maximize $R_1 = q_1[90 - 3(q_1 + q_2)]$ while firm 2 still chooses $q_2$ to maximize $\pi_2 = q_2[90 - 3(q_1 + q_2)] - 9q_2$. The Nash equilibrium is given by the solution to
\[ \frac{\partial R_i}{\partial q_1} = 0 \quad \text{and} \quad \frac{\partial \pi_2}{\partial q_2} = 0. \] The solution is \( q_1 = 11 \) and \( q_2 = 8 \) with corresponding profits \( \pi_1 = 264 \) and \( \pi_2 = 192 \).

(b.2) Since total output has increased (from 18 to 19) price is lower and thus consumers are better off.

(b.3) The owner of firm 2 is hurt by the change: her profits have decreased from 243 to 192.

(b.4) The owner of firm 1 earns more. The firm’s revenue is 363, thus the manager’s remuneration is 3.63. Hence the income of the owner of firm 1 increases from 243 to 264 – 3.63 = 260.37.

3.

(a) The game is as follows (C means “contribute” and N means “Not contribute”)

(b) There are four Nash equilibria, which are highlighted above: one where nobody contributes and three where exactly 2 people contribute.

(c) The equilibrium (N,N,N) is Pareto inefficient (it is Pareto dominated by any of the other equilibria), while the remaining equilibria are Pareto efficient.

(d) One equilibrium is where nobody contributes. All the other equilibria are those where exactly \( k \) people contribute. There is no equilibrium where more than \( k \) people contribute, because one of them can increase his payoff by switching to \( N \) (the party is still held and he saves money). There is no equilibrium where less than \( k \) people contribute, because any of the contributors can increase his payoff by switching to \( N \) (the party is not held in any case and he saves money).

(e) Of course we would have had the same set of Nash equilibria!