1. (a) The game-frame is as follows:

(b) At a subgame-perfect equilibrium Player 1 chooses $A$ at each of his second-stage nodes. Thus the game reduces to:
The strategic form associated with the reduced game is:

Using utility functions with values 0, 1, 2, we can write it as follows:

This game has 5 pure-strategy Nash equilibria: \((A,A,A)\), \((A,C,A)\), \((A,B,B)\), \((B,B,B)\) and \((C,C,C)\). Thus there are 5 subgame-perfect equilibria of the original game, obtained by appending to each first-round choice of Player 1 the choice of \(A\) at each of the six second-stage nodes.

2. (a) Let \(p\) be the probability that 002 uses strategy \(x\) (and \((1-p)\) the probability that he uses \(y\). Then, for 001, the expected payoffs from his pure strategies are: \(a(p) \rightarrow 12p\) \(b(p) \rightarrow 10p + 1\) \(c(p) \rightarrow 6p + 4\) \(d(p) \rightarrow 3p + 6\). The corresponding graphs are shown below:
Thus if \( p < \frac{2}{3} \) the best choice is d, if \( p = \frac{2}{3} \) the best choices are a, c and d, and if \( p > \frac{2}{3} \) then the best choice is a.

(a) Strategy b is never a best reply (for every value of \( p \) there is another pure strategy which is strictly better than b) and therefore should never be used.

(b) Based on the above, the following is a mixed-strategy Nash equilibrium: 001 chooses c (with probability 1) and 002 chooses x with probability \( \frac{2}{3} \) and y with probability \( \frac{1}{3} \):

\[
\begin{pmatrix}
0 & 0 & 1 & 0 \\
\frac{2}{3} & \frac{1}{3}
\end{pmatrix}.
\]

Since b must be given zero probability, to find an equilibrium where c is given zero probability, we must look for a mixture of a and d. Let q be the probability of a. Then for 002 x gives an expected payoff of \( 3(1-q) \) and y an expected payoff of \( 6q \). These two must be equal. Thus q must be \( \frac{1}{3} \). Then 002 is indifferent between any two mixed (or pure) strategies, in particular he will be happy to choose x with probability \( \frac{2}{3} \) and y with probability \( \frac{1}{3} \), in which case 001 is happy with any combination of a and d. Thus the following is a mixed-strategy equilibrium:

\[
\begin{pmatrix}
\frac{1}{3} & 0 & 0 & \frac{2}{3} \\
\frac{2}{3} & \frac{1}{3}
\end{pmatrix}.
\]

There is no other Nash equilibrium where Player 1 mixes between only two strategies (if Player 1’s mixture is between a and c, then Player 2 wants to play y with probability 1 and if Player 1’s mixture is between c and d, then Player 2 wants to play x with probability 1). Thus the only other candidates for NE are the profiles of the form

\[
\begin{pmatrix}
a & b & c & d \\
p & 0 & q & 1-p-q
\end{pmatrix}
\]

with \( p > 0, q > 0 \) and \( p + q < 1 \). In order for Player 2 to be indifferent between x and y it must be that Player 1 assigns to d two times the probability that he assigns to a. Thus all of the following are Nash equilibria:

\[
\begin{pmatrix}
a & b & c & d \\
\frac{1}{3}(1-q) & 0 & q & \frac{2}{3}(1-q)
\end{pmatrix}
\]

for any \( 0 < q < 1 \).