1. Consider the following game-frame in extensive form. Three players have to choose a candidate from the set \{Ann, Bob, Carla\}. They have agreed on the following voting procedure. There is a first round where each player votes for one candidate; voting in this round is **simultaneous** and secret (thus when a player writes down his/her vote, he/she does not know for which candidate the other players are voting). The votes are then revealed and if there is one candidate who has received at least two votes, then the game ends and that candidate is chosen; if every candidate received one vote then there is a second stage where Player 1 chooses the winner.

(a) Draw the game-frame in extensive form.

(b) Suppose that the players’ preferences are common knowledge and are as follows:

\[
\begin{align*}
Ann &>_{1} Carla >_{1} Bob, & Carla &>_{2} Bob >_{2} Ann, & Bob &>_{3} Ann >_{3} Carla
\end{align*}
\]

Find all the pure-strategy subgame-perfect equilibria. If you decide to use a utility function, use one that takes on values 0, 1 and 2.

2. The famous British spy 001 has to choose one of four routes to ski down a mountain: a, b, c or d (listed in order of speed in good conditions). Fast routes are more likely to be struck by an avalanche. At the same time, the notorious Russian rival spy 002 has to choose whether to use (y) or not use (x) his valuable explosive device to cause an avalanche. The von Neumann-Morgenstern payoffs of this game are given in the table below.

\[
\begin{array}{cccc}
 & x & y \\
 a & 12, 0 & 0, 6 \\
b & 11, 1 & 1, 5 \\
c & 10, 2 & 4, 2 \\
d & 9, 3 & 6, 0 \\
\end{array}
\]

(a) Let \( \mu(x) \) be the probability that 001 attaches to 002 selecting strategy x. Explain what 001 should do if \( \mu(x) > \frac{2}{3} \), if \( \mu(x) = \frac{2}{3} \) and if \( \mu(x) < \frac{2}{3} \).

(b) Imagine that you are Mr. Cue, the erudite technical advisor to British military intelligence. Are there any routes you would advise 001 definitely not to take? Explain your answer.

A viewer of this epic drama is trying to determine what will happen. Find a Nash equilibrium in which one player plays a pure strategy, call it \( s \), and the other player plays a mixed strategy. Find a different mixed-strategy equilibrium in which the above strategy \( s \) is assigned zero probability. Are there any other equilibria?