Spence's model of Signaling in the job market

I. Background.

The Arrow-Debreu model of a perfectly competitive economy is based on the crucial assumption of *complete markets*, that is, the assumption that there exists a market (i.e. a price) for each good, for each location, for each future date and for each possible state of nature. The presence of a complete set of forward markets for future contingent delivery eliminates uncertainty since it allows complete insurance. It is sometimes claimed that the properties of the Arrow-Debreu model (in particular, the results that a competitive equilibrium exists and that all competitive equilibria are Pareto efficient) depend crucially on the assumption of perfect information. This is not correct: it was proved by Radner in 1968 that - as long as the assumption of complete markets is retained - the presence of *asymmetric information* (some agents are more informed than others) is not an obstacle to the proof that a competitive equilibrium exists and is Pareto efficient.

The novelty of the market signaling literature, therefore, does not lie in the introduction of the hypothesis of asymmetric information, but rather in considering more complex situations where the set of signals is larger than that consisting of prices only. Models like Spence's are very limited in their scope and are not comparable to the Arrow-Debreu model, but their relevance lies in the fact that they show that the introduction of more complex (and closer to reality) informational assumptions may destroy one of the basic results of the Arrow-Debreu model: the Pareto efficiency of equilibria. A signaling equilibrium may not be Pareto efficient.

II. Education as a signal.

The situation considered by Spence, typified by the job market, is one where some relevant information is available to only one side of the market (the potential employees), while the other side of the market (the employer) has to try to infer this information from some observable characteristics. Spence assumes that the population is divided in two groups: those with low productivity and those with high productivity. The employer does not know in advance whether a given applicant belongs to one group or the other and it will take some time before the true productive ability of the employee is revealed. It is therefore in the employer's interest to try to guess the applicant's productive ability on the basis of some characteristics that the employer himself can observe. Those observable characteristics which are under the control of the applicant (like education, the way he dresses, etc.) are called ***signals,*** whereas those characteristics which cannot be modified by the applicant (e.g. sex, race, etc.) are called ***indexes.*** It is clear that not every observable characteristic will be considered relevant by the employer. For example, the color of the applicant's eyes (an index) or the color of his socks (a signal) are unlikely to be considered important by the employer, while the level of education is likely to be considered relevant. Therefore, within the set of potential signals the employer will select those that - *in the light of her previous experience* - seem to be correlated with productive capabilities. This selection represents the *beliefs* of the employer. Suppose the employer believes that education is positively correlated with productivity. Then she will offer a higher salary to those applicants who have acquired more education. We therefore have a *first flow of information,* *from the employer to the prospective employees*: by differentiating salaries on the basis of some characteristics the employer reveals her beliefs.

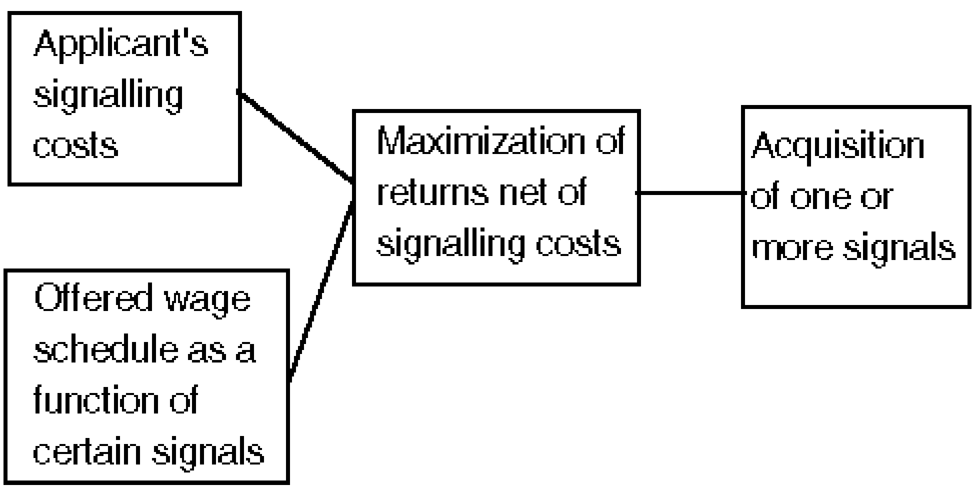
Employer's initial beliefs → Selection of signals within the set of all potential signals → Employees "read" the employer's beliefs in the wage schedule she offers

Let us now turn to the other side of the market: potential employees. A potential employee will face the problem of communicating his unobservable qualities to the employer. The only way he can do this is by using some observable characteristic that can be modified by him, that is, by using a signal. In choosing a signal the potential employee must take into account two factors:

1) *signaling costs*, that is, the cost of acquiring that particular signal;

2) *the employer's beliefs*, revealed by the wage schedule (it would be a waste of money and/or effort to acquire a signal which is known to be considered irrelevant by the employer, i.e. a characteristic to which the employer does not pay attention).

We therefore have a *second flow of information, from the applicant to the employer*:



At this point the wage contract will be stipulated and after a while the employer will be able to observe the employee's productivity and to relate it to the signal chosen by the employee. The employer's beliefs can then be confirmed (if, for example, the more educated employees are more productive) or falsified (if the more productive are the less educated employees, or if there is no difference in productivity between more and less educated employees). In the first case the employer will not modify her (beliefs and) offered wage schedule, while in the second case she will. We *define* the first case to be a ***signaling equilibrium***. That is, a signaling equilibrium is a situation in which employers' beliefs about the relationship between productivity (which cannot be observed at the time of hiring) and education (or any other signal) are confirmed by the results of her hiring in the market.

Thus in equilibrium employers' beliefs are *self-confirming*: the initial beliefs, based on some data (past experience), generate new data which does not contradict them and hence the beliefs tend to persist.

The simplest version of Spence's model is based on two main assumptions, one of which is crucial and seems to be necessary for the phenomenon of signaling to take place, whereas the second can be relaxed without affecting the qualitative nature of the results.

CRUCIAL ASSUMPTION. The crucial assumption is that *signaling costs are negatively correlated with productive ability*. That is, people with higher productivity incur lower costs of acquiring education (or the alternative relevant signal) than those with lower productivity. By 'costs of acquiring education' we do not necessarily mean monetary costs: it could simply be effort (e.g. measured in number of hours of study necessary to learn a particular thing).

RELAXABLE ASSUMPTION. The second assumption, which is not essential, is that education does not affect productivity. Spence himself developed a model in which this assumption is relaxed.

MAIN RESULTS.

The two main results of the model are:

(1) there may be multiple equilibria (and even an infinite number of them);

(2) equilibria may be (and in general are) Pareto inefficient.

The second result needs some explanation. First of all, it can easily be understood why the inefficiency occurs. Since the main hypothesis is that of asymmetric information it is not legitimate to compare the signaling equilibrium with a perfect information equilibrium. So the benchmark is represented by the situation where no signaling takes place and employers -- not being able to distinguish between more productive and less productive applicants and not having any elements on which to base a guess -- offer the same wage to every applicant, equal to the average productivity. Call this the *non-signaling equilibrium*. In a signaling equilibrium (where employers' beliefs are confirmed, since less productive people do not invest in education, while the more productive do) everybody may be worse off than in the non-signaling equilibrium. This occurs if the wage offered to the non-educated is lower than the average productivity (= wage offered to everybody in the non-signaling equilibrium) and that offered to the educated people is higher, but becomes lower (than the average productivity) once the costs of acquiring education are subtracted. The possible Pareto inefficiency of signaling equilibria is a strong result and a worrying one: it means that society is wasting resources in the production of education. However, it is not per se enough to conclude that education (i.e. the signaling activity) should be eliminated. The result is not that, *in general*, elimination of the signaling activity leads to a Pareto improvement: Spence simply pointed out that this is a *possibility*.

What *is* a general result is that within the set of possible equilibria (recall that multiple equilibria represent the rule and not the exception) some are Pareto superior to others. That is, *a Pareto ranking of the signaling equilibria is normally possible*. This is a strong result, because it goes against one of the main results of the Arrow-Debreu model. In the Arrow-Debreu model multiple equilibria are possible, but each equilibrium is Pareto efficient and therefore equilibria are not Pareto comparable.

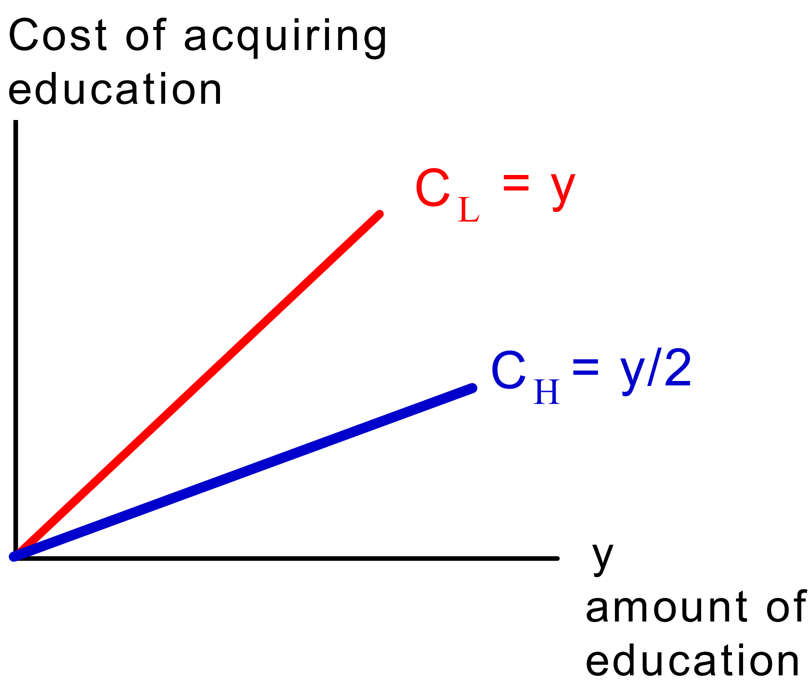
THE MODEL

Suppose that there are two groups of individuals (where $0 < q < 1$):

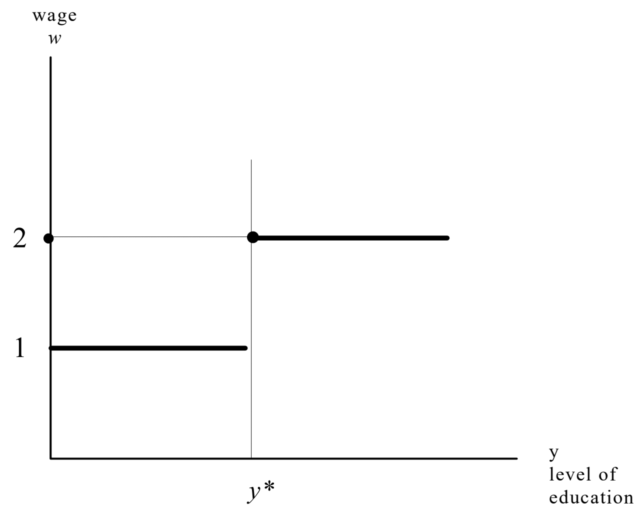
Group L	Group H
Marginal productivity = 1	Marginal productivity = 2
Proportion in population: q	Proportion in population: $1-q$

Note the implicit assumption that a person's productivity is constant, in particular it does not depend on the level of education acquired by the person.
Suppose that the labor market is perfectly competitive, so that in a situation of perfect information (where employers were able to distinguish low-productivity from high-productivity individuals) each worker would be paid a wage equal to his/her marginal productivity.
The *average productivity* over the entire population is: $1(q)+2(1-q)=2-q$ (>1 since $0<q<1$).
Suppose that workers of both types are able to buy education, at a cost. Assume that the amount of education y is a continuous variable and that it is fully verifiable (e.g. through a certificate). Suppose also that type-L workers face a higher cost of acquiring education than type-H workers. The cost is not necessarily a monetary cost: it could be a disutility cost (type-L individuals have to study harder and longer hours than type H individuals in order to obtain the same education level). Let
Cost of education for Group L individuals: $C_L = y$

Cost of education for Group H individuals: $C_H = \frac{y}{2}$



With the presence of education as a signal, employers will form beliefs about the correlation between education and productivity. For example, suppose that employers believe that anybody with a level of education less than y^* has a productivity of 1 (and thus are offered a wage of 1) while everybody with a level of education greater than or equal to y^* has a productivity of 2 (and thus are offered a wage of 2) . The employers' beliefs are thus reflected in the following wage schedule:



Given this wage schedule, it is clear that nobody will want to acquire a level of education $y > y^*$ or a level $0 < y < y^*$. Every individual will limit himself to choosing between $y = 0$ and $y = y^*$.

For a GROUP L individual	If choose $y = 0$	get $w = 1$ pay $C = 0$ net wage $= 1$
	If choose $y = y^*$	get $w = 2$ pay $C = y^*$ net wage $= 2 - y^*$
For a GROUP H individual	If choose $y = 0$	get $w = 1$ pay $C = 0$ net wage $= 1$
	If choose $y = y^*$	get $w = 2$ pay $C = y^*$ net wage $= 2 - \frac{y^*}{2}$

If y^* is such that $2 - y^* < 1$ and $2-\frac{y^*}{2}>1$, that is, if $\boxed{1<y^*<2}$ then group L individuals will choose $y = 0$ and group H individuals will choose $y = y^*$ and therefore the employers' beliefs will be confirmed: people with low education will turn out to be of low productivity and people with high education will turn out to have high productivity. This is called a ***separating signaling equilibrium.***

REMARKS.

(1) There is an ***infinity*** of such equilibria (one for each value of y^* strictly between 1 and 2).

(2) At any such equilibrium, Group L workers get a wage of 1 and have zero education costs, while Group H workers get a wage of 2 but face education costs equal to $\frac{y^*}{2}$. Since in this model education does not affect productivity and is of no value to the worker (it only involves a cost and no benefit), the lower the value of y^* the better off Group H workers are. Hence the equilibria can be ***Pareto ranked***.

(3) Can a separating signaling equilibrium be Pareto efficient? Let us compare an arbitrary separating signaling equilibrium with the non-signaling equilibrium in which everybody gets paid a wage equal to the average productivity of the entire population, namely $w = 2 - q$. Group L workers would be better off, since $2 - q > 1$ (because $q < 1$), so that they strictly prefer the non-signaling equilibrium. Group H workers prefer the non-signaling equilibrium if and only if $2-q>2-\frac{y^*}{2}$, that is, if and only if $\boxed{y^*>2q}$. For example, if $y^* = 1.5$ and $q = \frac{1}{2}$ this condition is satisfied and therefore everybody prefers the non-signaling equilibrium: if the government intervened and shut down all the schools, everybody would be better off! Thus for every y^* such that (1) $1 < y^* < 2$ and (2) $y^* > 2q$ we have a signaling equilibrium where Group L workers choose $y = 0$ and Group H workers choose $y = y^*$ and the equilibrium is strictly Pareto dominated by the non-signaling equilibrium where everybody is paid the average wage $w = 2 - q$.

III. The informational impact of indexes.

So far we only considered signals, that is, those observable characteristics that can be modified by the agents concerned. The observable characteristics which cannot be modified, like sex, race, nationality, etc., are called *indexes*. The question that the presence of indexes gives rise to is: can the informational structure of the market have the effect of persistently and consistently discriminating between "objectively" identical individuals? We can imagine that the population is divided into two groups of equal size: Whites and Blacks. Within each group we have an equal proportion of low-productivity and high-productivity individuals. Signaling (education) costs are different for low- and high-productivity people, but people with the same level of productivity have the same signaling costs, no matter whether they are white or black. Assume also that people with the same productivity have the same preferences and the same objective: to maximize their income net of signaling costs. Therefore the index (race, say) should be absolutely irrelevant: it is a general principle of economics that people with the same opportunity sets and the same preferences will make similar decisions and end up in similar situations. In our model people with the same level of productivity face the same maximization problem, with the same data, and therefore should make the same decisions concerning education, no matter whether they are white or black. The informational structure of the market, however, can destroy this principle. If the employer believes that race (besides education) is correlated with productivity, she might offer a wage schedule which is differentiated on the basis of race and education. Her beliefs may force high-productivity black people to invest in education more than their white counterpart, that is, more than the high-productivity Whites. The reason why this situation can persist is that employers will interpret the incoming data *separately* for the two groups of Whites and Blacks because they believe that race is a splitting factor. If different levels of education were associated with the same observed level of productivity *within the same group*, employers would be forced to revise their beliefs. That is, if within the group of Whites different levels of education were accompanied by the same observed productivity, then employers would conclude that, at least above a certain level, education no longer increases productivity. But since Blacks and Whites are judged separately and independently (data on Whites is not used to classify Blacks and *vice versa*) employers can consistently think that Blacks need to acquire more education than their white counterpart in order to compensate for a "genetic" handicap. As before, employers' beliefs may force Blacks to invest more in education than Whites, thereby confirming employers beliefs, despite the fact that those beliefs have no objective grounds. Thus Blacks end up being over-qualified for their jobs as compared to Whites.

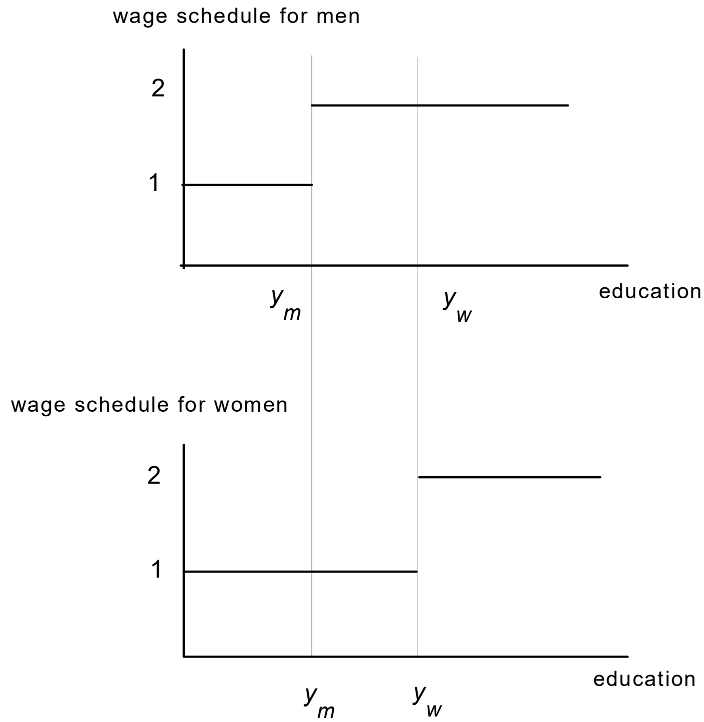
EXAMPLE. Suppose that the population is 50% men and 50% women. Within the male population, 50% are low-productivity (= 1) and 50% are high productivity (= 2). Similarly, within the female population, 50% are low-productivity (= 1) and 50% are high productivity (= 2). **Productivity** is innate and **is not influenced by either education or gender**.

	Women, L	Women, H	Men, L	Men, H
productivity	1	2	1	2
proportion	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$
Cost of acquiring y years of education	y	$\frac{y}{2}$	y	$\frac{y}{2}$

Suppose that employers mistakenly believe, not only that education affects productivity, but also that women are somewhat "genetically handicapped" and therefore need to acquire more education in order to become more productive. Accordingly, the employers offer the following wage schedules, with $y_w > y_m$:

If applicant is male		If applicant is female	
Education	Wage	Education	Wage
$y < y_m$	1	$y < y_w$	1
$y \geq y_m$	2	$y \geq y_w$	2

Graphically:



What choices will men and women make?

For a MAN of type L	If choose $y = 0$	get $w = 1$ pay $C = 0$ net wage = 1
	If choose $y = y_m$	get $w = 2$ pay $C = y_m$ net wage = $2 - y_m$
For a MAN of type H	If choose $y = 0$	get $w = 1$ pay $C = 0$ net wage = 1
	If choose $y = y_m$	get $w = 2$ pay $C = y_m$ net wage = $2 - \frac{y_m}{2}$

A separating equilibrium occurs if y_m is such that $2 - y_m < 1$ and $2 - \frac{y_m}{2} > 1$, that is, if

$\boxed{1 < y_m < 2}$ then men of type L will choose $y = 0$ and men of type H will choose $y = y_m$ and therefore the employers' beliefs will be confirmed: men with low education will turn out to be of low productivity and men with high education will turn out to have high productivity.

For a WOMAN of type L	If choose $y = 0$	get $w = 1$ pay $C = 0$ net wage = 1
	If choose $y = y_w$	get $w = 2$ pay $C = y_w$ net wage = $2 - y_w$
For a WOMAN of type H	If choose $y = 0$	get $w = 1$ pay $C = 0$ net wage = 1
	If choose $y = y_w$	get $w = 2$ pay $C = y_w$ net wage = $2 - \frac{y_w}{2}$

A separating equilibrium occurs if y_w is such that $2 - y_w < 1$ and $2 - \frac{y_w}{2} > 1$, that is, if

$\boxed{1 < y_w < 2}$ then women of type L will choose $y = 0$ and women of type H will choose $y = y_w$ and therefore the employers' beliefs will be confirmed: women with low education will turn out to be of low productivity and women with high education will turn out to have high productivity.

Now, if both conditions are satisfied, that is, if $\boxed{1 < y_m < y_w < 2}$, then employers' beliefs will be confirmed in every respect: in order for women to become more productive they need to spend more time in school. A man acquires a productivity of 2 by spending y_m years in school, while women acquire a productivity of 2 by spending more time in school: y_w years rather than y_m.