1. The core is the set of \((x_1, x_2, x_3)\) such that

\[
\begin{array}{c|c}
\hline
x_1 & v(\{1\}) = 10 \\
\hline
x_2 & v(\{2\}) = 6 \\
\hline
x_3 & v(\{3\}) = 8 \\
\hline
x_1 + x_2 & v(\{1,2\}) = 18 \\
\hline
x_1 + x_3 & v(\{1,3\}) = 24 \\
\hline
x_2 + x_3 & v(\{2,3\}) = 16 \\
\hline
x_1 + x_2 + x_3 & v(\{1,2,3\}) = 30 \\
\hline
\end{array}
\]

From (5) and (7) we get that \(x_2 \leq 6\). This, together with (2), gives

\[x_2 = 6. \quad (8)\]

From (7) and (8) we get that \(x_1 + x_3 = 24\) so that

\[x_3 = 24 - x_1. \quad (9)\]

From (4) and (8) we get that

\[x_1 \geq 12. \quad (10)\]

From (6) and (8) we get that \(x_3 \geq 10\) and this, together with (9) gives \(x_1 \leq 14\).

Thus the core is the set of triples \((x_1, x_2, x_3)\) such that \(12 \leq x_1 \leq 14, x_2 = 6\) and \(x_3 = 24 - x_1\).
2. The core is the set of \((x_1, x_2, x_3)\) such that

\[
\begin{array}{l}
\begin{array}{c}
\begin{aligned}
x_1 & \geq \nu\{1\} = 0 \\
x_2 & \geq \nu\{2\} = 0 \\
x_3 & \geq \nu\{3\} = 0 \\
x_1 + x_2 & \geq \nu\{1,2\} = 40 \\
x_1 + x_3 & \geq \nu\{1,3\} = 0 \\
x_2 + x_3 & \geq \nu\{2,3\} = 50 \\
x_1 + x_2 + x_3 & = \nu\{1,2,3\} = 50
\end{aligned}
\end{array}
\end{array}
\]

From (6) and (7) we get that \(x_1 \leq 0\). This, together with (1), gives

\[
x_1 = 0. \tag{8}
\]

From (7) and (8) we get that \(x_2 + x_3 = 50\) so that

\[
x_3 = 50 - x_2. \tag{9}
\]

From (4) and (8) we get that

\[
X_2 \geq 40. \tag{10}
\]

Thus the core is the set of triples \((x_1, x_2, x_3)\) such that \(x_1 = 0\), \(x_2 \geq 40\) and \(x_3 = 50 - x_2\).
3. The Shapley value is \( x_1 = 14, x_2 = 11, x_3 = 9 \) and is calculated as follows:

<table>
<thead>
<tr>
<th>( v({1}) )</th>
<th>( v({2}) )</th>
<th>( v({3}) )</th>
<th>( v({1,2}) )</th>
<th>( v({1,3}) )</th>
<th>( v({2,3}) )</th>
<th>( v({1,2,3}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>8</td>
<td>6</td>
<td>24</td>
<td>22</td>
<td>18</td>
<td>34</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>order</th>
<th>probability</th>
<th>player 1’s marginal contribution</th>
<th>player 2’s marginal contribution</th>
<th>player 3’s marginal contribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>123</td>
<td>1/6</td>
<td>10</td>
<td>14</td>
<td>10</td>
</tr>
<tr>
<td>132</td>
<td>1/6</td>
<td>10</td>
<td>12</td>
<td>12</td>
</tr>
<tr>
<td>213</td>
<td>1/6</td>
<td>16</td>
<td>8</td>
<td>10</td>
</tr>
<tr>
<td>231</td>
<td>1/6</td>
<td>16</td>
<td>8</td>
<td>10</td>
</tr>
<tr>
<td>312</td>
<td>1/6</td>
<td>16</td>
<td>12</td>
<td>6</td>
</tr>
<tr>
<td>321</td>
<td>1/6</td>
<td>16</td>
<td>12</td>
<td>6</td>
</tr>
</tbody>
</table>

| sum   |             | 84                               | 66                               | 54                               |

Shapley value

4. The Shapley value is \( x_1 = 115, x_2 = 85, x_3 = 60 \) and is calculated as follows:

<table>
<thead>
<tr>
<th>( v({1}) )</th>
<th>( v({2}) )</th>
<th>( v({3}) )</th>
<th>( v({1,2}) )</th>
<th>( v({1,3}) )</th>
<th>( v({2,3}) )</th>
<th>( v({1,2,3}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>80</td>
<td>60</td>
<td>30</td>
<td>180</td>
<td>160</td>
<td>120</td>
<td>260</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>order</th>
<th>probability</th>
<th>player 1’s marginal contribution</th>
<th>player 2’s marginal contribution</th>
<th>player 3’s marginal contribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>123</td>
<td>1/6</td>
<td>80</td>
<td>100</td>
<td>80</td>
</tr>
<tr>
<td>132</td>
<td>1/6</td>
<td>80</td>
<td>100</td>
<td>80</td>
</tr>
<tr>
<td>213</td>
<td>1/6</td>
<td>120</td>
<td>60</td>
<td>80</td>
</tr>
<tr>
<td>231</td>
<td>1/6</td>
<td>140</td>
<td>60</td>
<td>60</td>
</tr>
<tr>
<td>312</td>
<td>1/6</td>
<td>130</td>
<td>100</td>
<td>30</td>
</tr>
<tr>
<td>321</td>
<td>1/6</td>
<td>140</td>
<td>90</td>
<td>30</td>
</tr>
</tbody>
</table>

| sum   |             | 690                             | 510                             | 360                             |

Shapley value

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