Topic: Cooperative games
(core and Shapley value)

VERY IMPORTANT: do **not** look at the answers until you have made a VERY serious effort to solve the problem. If you turn to the answers to get clues or help, you are wasting a chance to test how well you are prepared for the exams. I will **not** give you more practice problems later on.

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1. Consider the following cooperative game: N = {1, 2, 3} and

$$v(\{1\}) = 10, \quad v(\{2\}) = 6, \quad v(\{3\}) = 8$$

$$v(\{1,2\}) = 18, \quad v(\{1,3\}) = 24, \quad v(\{2,3\}) = 16$$

$$v(\{1,2,3\}) = 30.$$

Find the core.

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2. Consider the following cooperative game: N = {1, 2, 3} and

$$v(\{1\}) = v(\{2\}) = v(\{3\}) = 0$$

$$v(\{1,2\}) = 40, \quad v(\{1,3\}) = 0, \quad v(\{2,3\}) = 50$$

$$v(\{1,2,3\}) = 50$$

Find the core.

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3. Consider the following cooperative game: N = {1, 2} and

$$v(\{1\}) = 2, \quad v(\{2\}) = 5, \quad v(\{1,2\}) = 8.$$

(a) Find the core.

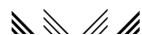**(b)** If imputations are required to be (component-by-component) integer-valued, list all the imputations in the core.

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4. Consider the following cooperative game: N = {1, 2, 3} and

$$v(\{1\}) = 4, \quad v(\{2\}) = 6, \quad v(\{3\}) = 3$$
$$v(\{1,2\}) = 14, \quad v(\{1,3\}) = 12, \quad v(\{2,3\}) = 16$$
$$v(\{1,2,3\}) = 18$$

For each of the following imputations (x_1, x_2, x_3) determine if it is in the core:

1. (6, 6, 6)
2. (4, 6, 8)
3. (7, 7, 4)
4. (8, 8, 2)

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5. Consider the following cooperative game: N = {1, 2, 3} and

$$v(\{1\}) = 2, \quad v(\{2\}) = 4, \quad v(\{3\}) = 1$$
$$v(\{1,2\}) = 12, \quad v(\{1,3\}) = 10, \quad v(\{2,3\}) = 14$$
$$v(\{1,2,3\}) = 16$$

Prove that the core is empty.

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6. Consider the following cooperative game: N = {1, 2, 3,4} and

$$v(\{1\}) = v(\{2\}) = 4, \quad v(\{3\}) = v(\{4\}) = 6$$
$$v(\{1,2\}) = v(\{1,3\}) = v(\{1,4\}) = 8, \quad v(\{2,3\}) = 10, \quad v(\{2,4\}) = 10, \quad v(\{3,4\}) = 12,$$
$$v(\{1,2,3\}) = v(\{1,2,4\}) = v(\{2,3,4\}) = 14,$$
$$v(\{1,2,3,4\}) = 18$$

For each of the following imputations (x_1, x_2, x_3, x_4) determine if it is in the core:

1. (4, 4, 5, 5)
2. (2, 4, 6, 6)
3. (4, 5, 5, 4)

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7. Consider the following cooperative game: N = {1, 2, 3,4} and

$$v(\{1\}) = v(\{2\}) = 4, \quad v(\{3\}) = v(\{4\}) = 6$$
$$v(\{1,2\}) = v(\{1,3\}) = v(\{1,4\}) = 8, \quad v(\{2,3\}) = 10, \quad v(\{2,4\}) = 10, \quad v(\{3,4\}) = 12,$$
$$v(\{1,2,3\}) = v(\{1,2,4\}) = v(\{2,3,4\}) = 14,$$
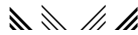$$v(\{1,2,3,4\}) = 18$$

Is the core non-empty?

8. Consider the following cooperative game: N = {1, 2, 3} and

$$v(\{1\}) = 10, \quad v(\{2\}) = 8, \quad v(\{3\}) = 6$$
$$v(\{1,2\}) = 24, \quad v(\{1,3\}) = 22, \quad v(\{2,3\}) = 18$$
$$v(\{1,2,3\}) = 34$$

Find the Shapley value.

9. Consider the following cooperative game: N = {1, 2, 3} and

$$v(\{1\}) = 80, \quad v(\{2\}) = 60, \quad v(\{3\}) = 30$$
$$v(\{1,2\}) = 180, \quad v(\{1,3\}) = 160, \quad v(\{2,3\}) = 120$$
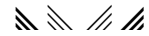$$v(\{1,2,3\}) = 260.$$

Find the Shapley value

10. Consider again the game of Exercise 9. Is Player 1 a dummy player?

11. Consider again the game of Exercise 9. Are Players 1 and 2 interchangeable?

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12. Consider the following cooperative game: N = {1, 2, 3} and

$$v(\{1\}) = 2, \quad v(\{2\}) = 4, \quad v(\{3\}) = 2$$
$$v(\{1,2\}) = 8, \quad v(\{1,3\}) = 10, \quad v(\{2,3\}) = 8$$
$$v(\{1,2,3\}) = 12$$

(a) Are Players 1 and 3 interchangeable?

(b) Find the Shapley value.

(c) Is the Shapley value in the core?

13. Consider the following cooperative game: N = {1, 2, 3} and

$$v(\{1\}) = 2, \quad v(\{2\}) = 4, \quad v(\{3\}) = 6$$
$$v(\{1,2\}) = 6, \quad v(\{1,3\}) = 8, \quad v(\{2,3\}) = 12$$
$$v(\{1,2,3\}) = 14$$

(a) Are any two players interchangeable?

(b) Is any player a dummy player?

(c) Find the Shapley value.

(d) Is the Shapley value in the core?