PRACTICE PROBLEMS $\boxed{4}$

Topic: Strategic voting

VERY IMPORTANT: do **not** look at the answers until you have made a VERY serious effort to solve the problem. If you turn to the answers to get clues or help, you are wasting a chance to test how well you are prepared for the exams. I will **not** give you more practice problems later on.

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1. Consider the following voting scheme: there are n voters ($n \geq 2$) and m alternatives ($m \geq n$). Each voter is asked to report his/her strict, complete and transitive preference ordering. Voter 1 is the first to do so. Her announcement is revealed to all the other voters. Now it is voter 2's turn to make his announcement, which is then revealed to all the other voters, and so on. The alternative is selected as follows: first the alternative which is at the *bottom* of 1's announced ordering is eliminated; of the remaining alternatives the one which is ranked *lowest* in 2's announced ordering is eliminated, and so on, up to player $n-1$. Finally, of the remaining alternatives, the one which is ranked *highest* in voter n's ordering is selected. Is this voting scheme strategy proof (or non-manipulable)? Prove your claim.

2. Let $W = \{x,y,z\}$ be a set of states, $\mathcal{P}$ the set of strict preference orderings of W and $f : \mathcal{P}^2 \to W$ the following voting scheme: if both individuals rank the same alternative at the top, then that alternative is chosen; otherwise the middle alternative of individual 1 is chosen, unless it is the lowest-ranked of individual 2, in which case the middle alternative of individual 2 is chosen.

(i) Represent this voting scheme by filling in the following table.

Individual 2's reported ranking

		x, y, z	x, z, y	y, x, z	y, z, x	z, x, y	z, y, x
individual	x, y, z						
1's	x, z, y						
reported	y, x, z						
ranking	y, z, x						
	z, x, y						
	z, y, x						

(ii) Show that this voting scheme is (a) nondictatorial and (b) not strategy-proof.

3. Consider the following voting scheme: there are three voters (1, 2 and 3) and three alternatives (*a, b* and *c*). Each voter reports a strict ranking of the three alternatives. First the pair (*a,b*) is considered and the winner of these two alternatives is the one which is ranked higher than the other by a majority of the three individuals. Then the winner between *a* and *b* is put up against *c* and the final winner is the one which is ranked higher than the other by a majority of the three individuals.

(a) Represent this voting scheme by filling in the following tables.

2's →	abc	acb	bac	bca	cab	cba
1's ↓					
abc					
acb					
bac					
bca					
cab					
cba					

3 reports abc

2's →	abc	acb	bac	bca	cab	cba
1's ↓					
abc					
acb					
bac					
bca					
cab					
cba					

3 reports acb

2's →	abc	acb	bac	bca	cab	cba
1's ↓					
abc					
acb					
bac					
bca					
cab					
cba					

3 reports bac

2's →	abc	acb	bac	bca	cab	cba
1's ↓					
abc					
acb					
bac					
bca					
cab					
cba					

3 reports bca

`	abc	acb	bac	bca	cab	cba
1's ↓					
abc					
acb					
bac					
bca					
cab					
cba					

3 reports cab

`	abc	acb	bac	bca	cab	cba
1's ↓					
abc					
acb					
bac					
bca					
cab					
cba					

3 reports cba

(b) Show that this voting scheme is not strategy proof.