Econ 102 (Analysis of Economic Data): Cameron Winter 2014
Solutions to Second Midterm Exam

Version A

1. (a) $\Delta \ln y = 7.5 - 3 \simeq 4.5$; $\Delta \text{year} \simeq 65$. Growth rate in levels $\simeq 4.5/65 \simeq 0.07$ or 7% per year.
   (b) $72/r$ years to double. Here $72/r = 4$ so $r = 18\%$ per annum.
   (c) $(y_t + y_{t-1} + y_{t-2} + y_{t-3})/4$.
   (d) FRED (Federal Reserve Bank of St. Louise economic database).
   (e) A line chart with both $y$ and $x$ plotted against year.
   (f) $b = (2 + 3 + 5 + 6)/4 = 4$ and $\bar{y} = (6 + 1 + 1 + 4)/4 = 3$.

   $x_i \ y_i \ x_i - \bar{x} \ y_i - \bar{y} \ (x_i - \bar{x})(y_i - \bar{y}) \ (x_i - \bar{x})^2$
   2 6 -2 3 -6 4
   3 1 -1 -2 2 1
   5 1 1 -2 -2 1
   6 4 2 1 2 4

   $b_2 = \sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y}) / \sum_{i=1}^n (x_i - \bar{x})^2 = -4/10 = -0.4$.  
   $b_1 = \bar{y} - b_2 \bar{x} = 3 - (-0.4) \times 4 = 4.6$.

2. (a) The mean charge increases by $1000b_2 = 1000 \times 1.314908 = 1,315$ dollars.
   (b) A 95% confidence interval for $\beta_2$ is $(1.0562, 1.5736)$ from the regression output.
   (c) A 99% confidence interval for $\beta_2$ is $b_2 \pm t_{0.05,169} \times s_{b_2}$
   $= 1.314908 \times \text{invttail}(167,.005) \times 1.3105 = 1.3149 \pm 2.6056 \times 1.3105 = 1.3149 \pm 3.415 = (.9734, 1.6564)$.
   (d) $H_0 : \beta_2 = 0$ against $H_a : \beta_2 \neq 0$.
   Using p-value approach: $p = 0.000 < 0.05$. So reject $H_0$.
   Alternatively and equivalently $|t| = 10.03 > t_{0.025,166} = \text{invttail}(167,.025) = 1.974$. So reject $H_0$.
   Conclude there is a statistically significant relationship at level 0.05.
   (e) $H_0 : \beta_2 \leq 0$ against $H_a : \beta_2 > 0$. Important: Claim is the alternative hypothesis.
   First $b_2 > 0$ so on the right side for an upper-tail alternative.
   Using p-value approach: $p = 0.000/2 = 0.000 < 0.05$. So reject $H_0$.
   Alternatively and equivalently $t = 10.03 > t_{0.025,166} = \text{invttail}(167,.05) = 1.654$. So reject $H_0$.
   Conclude that agree with claim that mean charge rises with mean cost at level 0.05.
   (f) $H_0 : \beta_2 = 1$ against $H_a : \beta_2 \neq 1$.
   $t = (b_2 - 1)/s_{b_2} = (1.3149 - 1)/.13105 = .3149/.13105 = 2.403$.
   Since $|t| = 2.403 < t_{0.05,166} = \text{invttail}(167,.05) = 2.606$ we do not reject $H_0$.
   Conclude that mean charge rises by $1$ for each $1$ increase in mean cost at level 0.01.

3. (a) $\bar{y} = b_1 + b_2 x = 20334.33 + 1.314908 \times 20000 = 846.632$.
   (b) A wrong answer (no credit) is $1/1.314908 = .7605$.
   A full credit answer is that there is not enough information.
   (A better answer, this is tricky, is that the slope coefficient from regress $y$ on $x$ is $r_{xy} \times s_y/s_x = \sqrt{.3761 \times 10376/22249} = .2860$.)
   (c) This is just the sample mean of $\text{meancharge}$: 47957.27.

4. (a) $R^2 = \text{ExplainedSS} / \text{TotalSS} = 40/160 = 0.25$.
   (b) Correlation coefficient $r_{xy} = \sqrt{R^2} = \sqrt{0.25} = 0.5$.
   (c) Standard error of the residual $= \sqrt{\text{ResidualSS} / (n-2)} = \sqrt{(160 - 40)/8} = \sqrt{15} = 3.87$.
   (d) This produces the residuals $y_i - \hat{y}_i$.

Multiple Choice for Versions A and B

Question 1. 2. 3. 4. 5.
Answer Version A d c a b a
Answer Version B a c a d b

For 3. the key assumptions are $y = \beta_1 + \beta_2 x + u$ and $E[u|x] = 0$.  

1
Version B

1. (a) \( \Delta \ln y = 9 - 3 \approx 6; \Delta \text{year} \approx 65 \). Growth rate in levels \( \approx 6/65 \approx 0.09 \) or 9% per year.
   
   (b) 72/r years to double. Here 72/r = 6 so r = 12% per annum.
   
   (c) \((y_t + y_{t-1} + y_{t-2} + y_{t-3} + y_{t-4})/5\).
   
   (d) FRED (Federal Reserve Bank of St. Louis economic database).
   
   (e) A scatter plot of y against x along with a regression line.

2. (a) The mean charge increases by 1000\( b_2 \) = 1000 \times 1.252196 = 1,252 dollars.
   
   (b) A 95% confidence interval for \( \beta_2 \) is (.9889, 1.5154) from the regression output.
   
   (c) A 90% confidence interval for \( \beta_2 \) is \( b_2 \pm t_{0.05,166} \times s_{b_2} \)
   
   (d) \( H_0 : \beta_2 = 0 \) against \( H_a : \beta_2 \neq 0 \).

Using p-value approach: \( p = 0.000 < 0.05 \). So reject \( H_0 \).
Alternatively and equivalently \(|t| = 9.39 > t_{.025,166} = \text{invttail}(166,.025) = 1.974 \) So reject \( H_0 \).
Conclude there is a statistically significant relationship at level 0.05.

(e) \( H_0 : \beta_2 \leq 0 \) against \( H_a : \beta_2 > 0 \). Important: Claim is the alternative hypothesis.
First \( b_2 > 0 \) so on the right side for an upper-tail alternative.

Using p-value approach: \( p = 0.000/2 = 0.000 < 0.05 \). So reject \( H_0 \).
Alternatively and equivalently \( t = 9.39 > t_{0.05,166} = \text{invttail}(166,.05) = 1.654 \). So reject \( H_0 \).
Conclude that agree with claim that mean charge increases with mean cost at level 0.05.

(f) \( H_0 : \beta_2 = 1 \) against \( H_a : \beta_2 \neq 1 \).

\( t = (b_2 - 1)/s_{b_2} = (1.2522 - 1)/.13333 = .2522/.13333 = 1.891 \).
Since \(|t| = 1.891 > t_{.05,167} = \text{invttail}(166,.05) = 1.654 \) we reject \( H_0 \).
Conclude that mean charge does not rise by $1 for each $1 increase in mean cost at level 0.10.

3. (a) \( \hat{y} = b_1 + b_2 x = 21432.8 + 1.2522 \times 30000 = $58,999 \).
   
   (b) A wrong answer (no credit) is 1/1.2522 = .7986.

A full credit answer is that there is not enough information.
(A better answer, this is tricky, is that the slope coefficient from regress y on x is \( r_{xy} \times s_y/s_x = \sqrt{.3470 \times 10132/21539} = .2771 \).
   
   (c) This is just the sample mean of meancharge: 47509.81.

4. (a) \( R^2 = \text{RegressionSS / TotalSS} = 10/250 = 0.04 \).
   
   (b) Correlation coefficient \( r_{xy} = \sqrt{R^2} = \sqrt{0.04} = 0.2 \).
   
   (c) Standard error of the residual = \( \sqrt{\text{ResidualSS / (n-2)}} = \sqrt{(250 - 10)/8} = \sqrt{30} = 5.48 \).
   
   (d) This produces the residuals \( y_i - \hat{y}_i \).

The course grade will be based on a curve from the combined scores of midterm 1 (22.5%), midterm 2 (22.5%), final (45%) and assignments (10%). The curve for this exam is only a guide.
Suggested average GPA for this course is 2.4. Curve below has average GPA 2.39 for this exam.

Scores out of 100

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<th>B+ 23</th>
<th>B 22</th>
<th>C- 18</th>
<th>C 19</th>
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A+ 27 and above C+ 20 and above
A 25 and above C 19 and above
B+ 23 and above D+ 17 and above
B 22 and above D 16 and above
B- 21 and above D- 14.5 and above