4-4 THIS QUESTION HAD SEVERAL TYPOS (in parts (c)-(e)). USE THE FOLLOWING CORRECTED QUESTION INSTEAD.

Consider the linear regression model with scalar regressor $y_i = \beta \mathbf{x}_i + u_i$ with data (y_i, x_i) iid over i though the error may be conditionally heteroskedastic.

(a) Show that $(\widehat{\beta}_{\mathrm{OLS}} - \beta) = (N^{-1}\sum_i x_i^2)^{-1} N^{-1}\sum_i x_i u_i$.

(b) Apply Kolmogorov law of large numbers (Theorem A.8) to the averages of x_i^2 and $x_i u_i$ to show that $\widehat{\beta}_{\mathrm{OLS}} \xrightarrow{p} \beta$. State any additional assumptions made on the dgp for x_i and u_i.

(c) Apply the Lindeberg-Levy central limit theorem (Theorem A.14) to the averages of $x_i u_i$ to show that $N^{-1}\sum_i x_i u_i / \sqrt{N^{-2}\sum_i \mathrm{E}[u_i^2 x_i^2]} \xrightarrow{d} \mathcal{N}[0, 1]$. State any additional assumptions made on the dgp for x_i and u_i.

(d) Use the product limit normal rule (Theorem A.17) to show that part (c) imples $N^{-1/2}\sum_i x_i u_i \xrightarrow{d} \mathcal{N}[0,\ \lim N^{-1}\sum_i \mathrm{E}[u_i^2 x_i^2]]$. State any assumptions made on the dgp for x_i and u_i.

(e) Combine results using (2.14) and the product limit normal rule (Theorem A.17) to obtain the lmit distribution of $\widehat{\beta}_{\mathrm{OLS}}$.