4-7 THIS QUESTION HAD SEVERAL ERRORS (notable (d)-(f)). USE THE FOLLOWING REVISED QUESTION INSTEAD.

(Adapted from Nelson and Startz, 1990). Consider the three equation model, $y = \beta x + u$; $x = \lambda u + \varepsilon$; $z = \gamma\varepsilon + v$, where the mutually independent errors u, ε and v are iid normal with mean 0 and variances, respectively, σ_u^2, σ_ε^2 and σ_v^2.

(a) Show that $\text{plim}(\widehat{\beta}_{\text{OLS}} - \beta) = \lambda\sigma_u^2 / \left(\lambda^2\sigma_u^2 + \sigma_\varepsilon^2\right)$.

(b) Show that $\rho_{XZ}^2 = [\gamma\sigma_\varepsilon^2]^2 / [(\lambda^2\sigma_u^2 + \sigma_\varepsilon^2)(\gamma^2\sigma_\varepsilon^2 + \sigma_v^2)]$.

(c) Show that $\widehat{\beta}_{\text{IV}} - \beta = m_{zu} / \left(\lambda m_{zu} + m_{z\varepsilon}\right) \xrightarrow{p} 0$, where, for example, $m_{zu} = N^{-1}\sum_i z_i u_i$.

(d) Show that $\widehat{\beta}_{\text{IV}} - \beta$ is not defined if $m_{zu} = -m_{z\varepsilon}/\lambda$. Nelson and Startz (1990) argue that this region is visited often enough that the mean of $\widehat{\beta}_{\text{IV}}$ does not exist.

(e) Show that $\widehat{\beta}_{\text{IV}} - \beta = 1/(\lambda + m_{z\varepsilon}/m_{zu})$ equals $1/\lambda$ if m_{zu} is large relative to $m_{z\varepsilon}/\lambda$. Nelson and Startz (1990) conclude that if m_{zu} is large relative to $m_{z\varepsilon}/\lambda$ then $\widehat{\beta}_{\text{IV}} - \beta$ is concentrated around $1/\lambda$, rather than the probability limit of zero from part (c).

(f) Nelson and Startz (1990) argue that $\widehat{\beta}_{\text{IV}} - \beta$ concentrates on $1/\lambda$ more rapidly the smaller is γ, the smaller is σ_ε^2, and the larger is λ. Given your answer in part (c), what do you conclude about the small sample distribution of $\widehat{\beta}_{\text{IV}}$ when ρ_{XZ}^2 is small?