

Estimating user-defined nonlinear regression models in Stata and in Mata

A. Colin Cameron
Univ. of Calif. - Davis

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Based on A. Colin Cameron and Pravin K. Trivedi,
Microeconometrics using Stata, Stata Press.

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1. Introduction

- Consider nonlinear cross-section regression of y_i on \mathbf{x}_i .
- Example is $y_i | \mathbf{x}_i \sim \text{Poisson}$ with mean $\mu_i = \exp(\mathbf{x}'_i \boldsymbol{\beta})$.
- This talk demonstrates various ways to code up the estimator,
 - using Stata command `m1`
 - and Mata command `optimize`

Outline

- ① Introduction
- ② Built-in command poisson
- ③ Command `m1` method `lf`
- ④ Checking program by simulation
- ⑤ Command `m1` methods `d0`, `d1`, `d2`
- ⑥ Newton-Raphson algorithm in Mata
- ⑦ Mata command `optimize`
- ⑧ NL2SLS example

2. Built-in command poisson

- Data from 2002 U.S. Medical Expenditure Panel Survey (MEPS).
Data due to Deb, Munkin and Trivedi (2006)
- Aged 25-64 years working in private sector but not self-employed and
not receiving public insurance (Medicare and Medicaid)
- Model `docvis` - annual number of doctor visits.

```
. use mus10data.dta, clear
. quietly keep if year02==1
. describe docvis private chronic female income
```

variable	storage type	display format	value label	variable label
docvis	int	%8.0g		number of doctor visits
private	byte	%8.0g		= 1 if private insurance
chronic	byte	%8.0g		= 1 if a chronic condition
female	byte	%8.0g		= 1 if female
income	float	%9.0g		Income in \$ / 1000

```
. summarize docvis private chronic female income
```

Variable	Obs	Mean	Std. Dev.	Min	Max
docvis	4412	3.957389	7.947601	0	134
private	4412	.7853581	.4106202	0	1
chronic	4412	.3263826	.4689423	0	1
female	4412	.4718948	.4992661	0	1
income	4412	34.34018	29.03987	-49.999	280.777

Built-in command poisson

```
. poisson docvis private chronic female income, vce(robust)
```

Iteration 0: log pseudolikelihood = -18504.413

Iteration 1: log pseudolikelihood = -18503.549

Iteration 2: log pseudolikelihood = -18503.549

Poisson regression

		Number of obs	=	4412
		wald chi2(4)	=	594.72
		Prob > chi2	=	0.0000
		Pseudo R2	=	0.1930

Log pseudolikelihood = -18503.549

docvis	Coef.	Robust Std. Err.	z	P> z	[95% Conf. Interval]
private	.7986652	.1090014	7.33	0.000	.5850263 1.012304
chronic	1.091865	.0559951	19.50	0.000	.9821167 1.201614
female	.4925481	.0585365	8.41	0.000	.3778187 .6072774
income	.003557	.0010825	3.29	0.001	.0014354 .0056787
_cons	-.2297262	.1108732	-2.07	0.038	-.4470338 -.0124186

Note: Nonrobust standard errors are (erroneously) much smaller.

Marginal effects for nonlinear model: $\partial E[y|\mathbf{x}]/\partial x_j = \beta_j \times \exp(\mathbf{x}'\boldsymbol{\beta})$.

. mfx

Marginal effects after poisson

y = predicted number of events (predict)
= 3.0296804

variable	dy/dx	Std. Err.	z	P> z	[95% C.I.]	x
private*	1.978178	.20441	9.68	0.000	1.57755	2.37881	.785358	
chronic*	4.200068	.27941	15.03	0.000	3.65243	4.7477	.326383	
female*	1.528406	.17758	8.61	0.000	1.18036	1.87645	.471895	
income	.0107766	.00331	3.25	0.001	.00428	.017274	34.3402	

(*) dy/dx is for discrete change of dummy variable from 0 to 1

. margeff

Average marginal effects on E(docvis) after poisson

docvis	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]
private	2.404721	.2438573	9.86	0.000	1.926769 2.882672
chronic	4.599174	.2886176	15.94	0.000	4.033494 5.164854
female	1.900212	.2156694	8.81	0.000	1.477508 2.322917
income	.0140765	.004346	3.24	0.001	.0055585 .0225945

3. Command ml method If

- First write a program we call `lfpois`.

This constructs the log-likelihood

$$\sum_{i=1}^N \ln f(y_i | \mathbf{x}_i, \boldsymbol{\beta}) = \sum_{i=1}^N \{-\exp(\mathbf{x}'_i \boldsymbol{\beta}) + y_i \mathbf{x}'_i \boldsymbol{\beta} - \ln y_i!\}.$$

- Then give commands

- `ml model lf lfpois (docvis = private chronic female income), vce(robust)`
- `ml check`
- `ml search`
- `ml maximize`

- The `ml check` and `ml search` are optional.

- ① y is stored in global macro `ML_y1`.
It is referred to as `$ML_y1`
- ② x is combined with β as the index $x'\beta$
It is referred to as the program argument `theta1`
- ③ $\ln f(y|x, \beta)$ is referred to as the program argument `lf`

```
. program define lfpois
1. version 10.0
2. args lnf theta1                      // theta1=x'b, lnf=lnf(y)
3. tempvar lnyfact mu
4. local y "$ML_y1"                      // Define y so program more readable
5. generate double `lnyfact' = lnfactorial(`y')
6. generate double `mu' = exp(`theta1')
7. quietly replace `lnf' = -`mu' + `y'^*`theta1' - `lnyfact'
8. end
```

Arguments, temporary variables and local variables are local macros,
referenced in single quotes.

Stata command ml method If for Poisson MLE

```
. * Compute the estimator  
. ml maximize
```

```
initial: log pseudolikelihood = -23017.072  
rescale: log pseudolikelihood = -23017.072  
Iteration 0: log pseudolikelihood = -23017.072  
Iteration 1: log pseudolikelihood = -19777.405  
Iteration 2: log pseudolikelihood = -18513.54  
Iteration 3: log pseudolikelihood = -18503.556  
Iteration 4: log pseudolikelihood = -18503.549  
Iteration 5: log pseudolikelihood = -18503.549
```

```
Number of obs      =      4412  
Wald chi2(4)      =      594.72  
Prob > chi2        =     0.0000  
Log pseudolikelihood = -18503.549
```

docvis	Robust					
	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
private	.7986654	.1090015	7.33	0.000	.5850265	1.012304
chronic	1.091865	.0559951	19.50	0.000	.9821167	1.201614
female	.4925481	.0585365	8.41	0.000	.3778187	.6072775
income	.003557	.0010825	3.29	0.001	.0014354	.0056787
_cons	-.2297263	.1108733	-2.07	0.038	-.4470339	-.0124188

- Command `ml` is not restricted to likelihood functions.
e.g. For OLS maximize $-\sum_{i=1}^N (y_i - \mathbf{x}'_i \boldsymbol{\beta})^2$.
`quietly replace 'lnf' = -('y'-exp('theta1'))^2`
But must then use robust standard errors.
- Command `ml` can handle models with more than one index.
e.g. For negative binomial have two indexes $\mathbf{x}'_i \boldsymbol{\beta}$ and α .
`args lnf theta1 a`
and
`ml model lf lfnb (docvis = private chronic female income) ()`
- Number of numerical derivatives = number of indexes.
Fast if few indexes.

4. Check program by simulation

- Generate sample of size N from

$$\begin{aligned}y_i &\sim \text{Poisson}[\exp(\alpha + \beta x_i)] \\x_i &\sim N[0, 0.5^2] \\\alpha &= 2; \beta = 1.\end{aligned}$$

- To check consistency
 - Set $N = 100,000$
 - Does $\hat{\alpha} = 2$? Does $\hat{\beta} = 1$?

- To check computation of the standard errors $s_{\hat{\alpha}}$ and $s_{\hat{\beta}}$.
 - Set $N = 500$.
 - Draw 2,000 samples of size N and obtain 2,000 estimates using command `simulate` or command `postfile`
 - Does $\sqrt{\frac{1}{1999} \sum_{s=1}^{2000} (\hat{\beta}^{(s)} - \bar{\hat{\beta}})^2} = \frac{1}{2000} \sum_{s=1}^{2000} s_{\hat{\beta}}^{(s)}$?
 - i.e. Over the simulations:
Does the st. deviation of $\hat{\beta}$ = the average st. error of $\hat{\beta}$?

5. Command ml methods d0, d1, d2

- More general.
- Computes the log-density for each observation.
This then needs to be summed using `mlsum`
- Enters parameters β directly, rather than via index $x'\beta$.
- Method d0 needs to compute q numerical derivatives if q parameters.
- Can provide first derivatives (method d1) and second derivatives (method d2).
This speeds up computation.

- For method d0 extra arguments is todo
- mleval converts β to $\mathbf{x}'\beta$
- mlsum converts $\mathbf{x}_i'\beta$ to $\sum_{i=1}^N \mathbf{x}_i'\beta$.

```
. * Method d0: Program d0opois to be called by command ml method d0
. program define d0opois
1. version 10.0
2. args todo b lnf                                // todo is not used, b=b, lnf=lnL
3. tempvar theta1                                 // theta1=x'b given in eq(1)
4. mleval `theta1' = `b', eq(1)
5. local y $ML_y1                                // Define y so program more readable
6. mlsum lnf' = -exp(`theta1') + `y'*`theta1' - lnfactorial(`y')
7. end
```

Stata command ml method d0 for Poisson MLE

```
. ml model d0 d0pois (docvis = private chronic female income)  
. ml maximize
```

```
initial:    log likelihood = -33899.609  
alternative: log likelihood = -28031.767  
rescale:    log likelihood = -24020.669  
Iteration 0: log likelihood = -24020.669  
Iteration 1: log likelihood = -18845.464  
Iteration 2: log likelihood = -18510.257  
Iteration 3: log likelihood = -18503.552  
Iteration 4: log likelihood = -18503.549  
Iteration 5: log likelihood = -18503.549
```

```
Number of obs      =      4412  
Wald chi2(4)      =     8052.34  
Prob > chi2       =     0.0000  
Log likelihood = -18503.549
```

docvis	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]
private	.7986653	.027719	28.81	0.000	.7443371 .8529936
chronic	1.091865	.0157985	69.11	0.000	1.060901 1.12283
female	.4925481	.0160073	30.77	0.000	.4611744 .5239218
income	.003557	.0002412	14.75	0.000	.0030844 .0040297
_cons	-.2297263	.0287022	-8.00	0.000	-.2859815 -.173471

Stata command ml method d2 for Poisson MLE

- Preceding gives nonrobust standard errors.
- To get robust standard errors need to use method d1 or d2.

```
. * Method d2: Program d2pois to be called by command ml method d2
. program define d2pois
1. version 10.0
2. args todo b lnf g negH          // Add g and negH to the arguments list
3. tempvar theta1                // theta1 = x'b where x given in eq(1)
4. mleval `theta1' = `b', eq(1)
5. local y $ML_y1                 // Define y so program more readable
6. mlsum `lnf' = -exp(`theta1') + `y'*`theta1' - lnsfactorial(`y')
7. if (`todo'==0 | `lnf'>=.) exit // d1 extra code from here
8. tempfile d1
9. mlvecsum `lnf' `d1' = `y' - exp(`theta1')
10. matrix `g' = (`d1')
11. if (`todo'==0 | `lnf'>=.) exit // d2 extra code from here
12. tempfile d11
13. mllmatsum `lnf' `d11' = exp(`theta1')
14. matrix `negH' = `d11'
15. end
```

6. Newton-Raphson algorithm using Mata

- Iterative algorithms are rules to compute $\hat{\theta}_{s+1}$ given $\hat{\theta}_s$.
- Gradient methods use a rule of the form

$$\hat{\theta}_{s+1} = \hat{\theta}_s + \mathbf{A}_s \mathbf{g}_s$$

where \mathbf{g}_s is the gradient of the objective function evaluated at $\hat{\theta}_s$.

- Newton-Raphson (NR) method approximates the objective function at $\hat{\theta}_s$ by a quadratic function.
It chooses $\hat{\theta}_{s+1}$ to maximize this approximation.

Then

$$\hat{\theta}_{s+1} = -\mathbf{H}_s^{-1} \mathbf{g}_s$$

where \mathbf{H}_s is the Hessian evaluated at $\hat{\theta}_s$.

- Poisson objective function, gradient and Hessian are:

$$\begin{aligned} Q(\beta) &= \sum_{i=1}^N \{-\exp(\mathbf{x}'_i \beta) + y_i \mathbf{x}'_i \beta - \ln y_i!\} \\ \mathbf{g}(\beta) &= \sum_{i=1}^N (y_i - \exp(\mathbf{x}'_i \beta)) \mathbf{x}_i \\ \mathbf{H}(\beta) &= \sum_{i=1}^N -\exp(\mathbf{x}'_i \beta) \mathbf{x}_i \mathbf{x}'_i. \end{aligned}$$

- So NR is

$$\begin{aligned} \hat{\beta}_{s+1} &= \hat{\beta}_s - \mathbf{H}(\hat{\beta}_s)^{-1} \times \mathbf{g}(\hat{\beta}_s) \\ &= \hat{\beta}_s + \left[\sum_{i=1}^N \exp(\mathbf{x}'_i \hat{\beta}_s) \mathbf{x}_i \mathbf{x}'_i \right]^{-1} \times \sum_{i=1}^N (y_i - \exp(\mathbf{x}'_i \hat{\beta}_s)) \mathbf{x}_i. \end{aligned}$$

- Core Mata code is

```
> mata
>   cha = 1                                // initialize stopping criterion
>   do {
>     mu = exp(X*b)
>     grad = X' (y-mu)                      // kx1 gradient vector
>     hes = makesymmetric((X:*mu)'X) // negative of the kxk hessian matrix
>     bold = b
>     b = bold + cholinv(hes)*(grad)
>     cha = (bold-b)'(bold-b)/(b'b)
>     iter = iter + 1
>   } while (cha > 1e-16)                  // end of iteration loops
> end
```

- ① Define y and x in Stata

```
generate cons = 1  
local y docvis  
local xlist private chronic female income cons
```

- ② Read these in to Mata using `st_view`

```
: st_view(y=., ., "'y'")  
: st_view(X=., ., tokens("'xlist'"))
```

- ③ Do the analysis in Mata and compute b and V

- ④ Pass these back to Stata using `st_matrix`

```
st_matrix("b",b')  
st_matrix("V",vb)
```

Post results using command `ereturn`

Do the NR iterations to compute $\hat{\beta}$.

```
. * Complete Mata code for Poisson MLE NR iterations
. mata
: st_view(y=., ., "`y'")           // read in stata data to y and x
: st_view(X=., ., tokens("`xlist'"))
: b = J(cols(X),1,0)               // compute starting values
: n = rows(X)
: iter = 1                         // initialize number of iterations
: cha = 1                           // initialize stopping criterion
: do {
>     mu = exp(X*b)
>     grad = X'(y-mu)              // kx1 gradient vector
>     hes = makesymmetric((X:*mu)'X) // negative of the kxk hessian matrix
>     bold = b
>     b = bold + cholinv(hes)*(grad)
>     cha = (bold-b)'(bold-b)/(b'b)
>     iter = iter + 1
> } while (cha > 1e-16)            // end of iteration loops
```

Compute the variance-covariance matrix of $\hat{\beta}$.

```
: mu = exp(x*b)
: hes = (x:*mu)'x
: vgrad = ((x:*(y-mu))'(x:*(y-mu)))
: vb = cholinv(hes)*vgrad*cholinv(hes)*n/(n-cols(x))
: iter // num iterations
13
: cha // stopping criterion
1.11465e-24
: st_matrix("b",b') // pass results from Mata to Stata
: st_matrix("V",vb) // pass results from Mata to Stata
: end
```

Present results nicely formatted.

```
. * Present results, nicely formatted using Stata command ereturn  
. matrix colnames b = `xlist'  
. matrix colnames v = `xlist'  
. matrix rownames v = `xlist'  
. ereturn post b v  
. ereturn display
```

	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]
private	.7986654	.1090509	7.32	0.000	.5849295 1.012401
chronic	1.091865	.0560205	19.49	0.000	.9820669 1.201663
female	.4925481	.058563	8.41	0.000	.3777666 .6073295
income	.003557	.001083	3.28	0.001	.0014344 .0056796
cons	-.2297263	.1109236	-2.07	0.038	-.4471325 -.0123202

7. Mata command optimize

- Mata command `optimize` uses same optimizer as command `m1`, but different syntax.
- Minimal syntax is

void evaluator(todo, p, v, g, H)

where

p is parameter vector

v defines objective function, and

if *todo* = 0 then gradient *g* and Hessian *H* are optional.

- Type *v* evaluator provides formula for $1 \times N$ vector *v*, where
 $\mathbf{e}'\mathbf{v} = f(\mathbf{p})$.
Suited to m-estimators (MLE, LS, just-identified NLIV).
- Type *d* evaluator provides formula for scalar *v* where $v = f(\mathbf{p})$.
Suited to over-identified generalized method of moments (GMM).

Declare the function poissonmle and st_view data

```
. mata
: void poissonmle(todo, b, y, x, lndensity, g, H) mata (type end to exit) --
> {
>     xb = x*b'
>     mu = exp(xb)
>     lndensity = -mu + y:*xb - lnfactorial(y)
>     if (todo == 0) return
>     g = (y-mu):*x
>     if (todo == 1) return
>     H = - cross(x, mu, X)
> }
: st_view(y=., ., "`y'")
: st_view(x=., ., tokens("`xlist'"))
```

Initialize command `optimize` and `optimize` using v2 evaluator.

```
: s = optimize_init()  
: optimize_init_evaluator(s, &poissonmle())  
: optimize_init_evaluatortype(s, "v2")  
: optimize_init_argument(s, 1, y)  
: optimize_init_argument(s, 2, x)  
: optimize_init_params(s, J(1,cols(x),0))  
:  
: b = optimize(s)  
Iteration 0: f(p) = -33899.609  
Iteration 1: f(p) = -19668.697  
Iteration 2: f(p) = -18585.609  
Iteration 3: f(p) = -18503.779  
Iteration 4: f(p) = -18503.549  
Iteration 5: f(p) = -18503.549
```

Compute variance covariance matrix and list results.

```
: vbrob = optimize_result_v_robust(s)
: serob = (sqrt(diagonal(vbrob)))'
: b \ serob
      1           2           3           4           5
1  .7986653788  1.091865108  .4925480693  .0035570127 - .2297263376
2  .1090014507  .0559951312  .0585364746  .0010824894  .1108732568
: end
```

Note: Can `st_matrix` back to Stata and `ereturn display` results.
Results are the same as from command Poisson.

8. NL2SLS example

- Poisson MLE inconsistent if $E[y - \exp(\mathbf{x}'\beta)|\mathbf{x}] \neq 0$, due to endogenous regressors.
- Assume there are instruments \mathbf{z} such that

$$E[\mathbf{z}_i(y_i - \exp(\mathbf{x}'\beta))] = 0.$$

- Define the $r \times 1$ vector

$$\mathbf{h}(\beta) = \left[\sum_i \mathbf{z}_i(y_i - \exp(\mathbf{x}'_i\beta)) \right].$$

- In just-identified case: # instruments = # regressors ($r = K$)
use the nonlinear instrumental variables (NLIV) estimator that solves

$$\mathbf{h}(\hat{\beta}) = \mathbf{0}.$$

- In over-identified case ($r > K$) the GMM estimator minimizes

$$Q(\beta) = \mathbf{h}(\beta)' \mathbf{W} \mathbf{h}(\beta).$$

- GMM estimator minimizes

$$Q(\beta) = \mathbf{h}(\beta)' \mathbf{W} \mathbf{h}(\beta).$$

- The $K \times 1$ gradient vector is

$$\mathbf{g}(\beta) = \partial Q(\beta) / \partial \beta = \mathbf{G}(\beta)' \mathbf{W} \mathbf{h}(\beta).$$

- The $K \times K$ expected Hessian is

$$\mathbf{H}(\beta) = \partial^2 Q(\beta) / \partial \beta \partial \beta' = \mathbf{G}(\beta)' \mathbf{W} \mathbf{G}(\beta)'.$$

- Where

$$\begin{aligned}\mathbf{G}(\beta) &= -\sum_i \exp(\mathbf{x}'_i \beta) \mathbf{z}_i \mathbf{x}'_i \\ \mathbf{h}(\beta) &= \sum_i \mathbf{z}_i (y_i - \exp(\mathbf{x}'_i \beta)) \\ \mathbf{W} &= (\mathbf{Z}' \mathbf{Z})^{-1} = \left(\sum_i \mathbf{z}_i \mathbf{z}'_i \right)^{-1}\end{aligned}$$

- Application treats private as endogenous with single instrument firmsize:

```
local zlist firmsize chronic female income cons
```

Declare the function pgmm and st_view data

```
. mata
: void pgmm(todo, b, y, x, z, Qb, g, H)                               mata (type end to exit)
> {
>     xb = x*b'
>     mu = exp(xb)
>     h = z'(y-mu)
>     w = cholinv(cross(z,z))
>     Qb = h'w*h
>     if (todo == 0) return
>     G = -(mu:_z)'x
>     g = (G'w*h)'
>     if (todo == 1) return
>     H = G'w*G
>     _makesymmetric(H)
> }
: st_view(y=., ., "`y'")
: st_view(x=., ., tokens("`xlist'"))
: st_view(z=., ., tokens("`zlist'"))
```

Initialize command optimize and optimize using d2 evaluator.

```
: s = optimize_init()  
: optimize_init_which(s,"min")  
: optimize_init_evaluator(s, &pgmm())  
: optimize_init_evaluatortype(s, "d2")  
: optimize_init_argument(s, 1, y)  
: optimize_init_argument(s, 2, x)  
: optimize_init_argument(s, 3, z)  
: optimize_init_params(s, j(1,cols(x),0))  
: optimize_init_technique(s,"nr")  
  
: b = optimize(s)  
Iteration 0: f(p) = 71995.212  
Iteration 1: f(p) = 9259.0408  
Iteration 2: f(p) = 1186.8103  
Iteration 3: f(p) = 3.4395408  
Iteration 4: f(p) = .00006905  
Iteration 5: f(p) = 5.672e-14  
Iteration 6: f(p) = 1.953e-27
```

Compute variance covariance matrix (manually) and list results.

```
: // Compute robust estimate of VCE and se's
: xb = x*b'

: mu = exp(xb)

: h = z'(y-mu)

: w = cholinv(cross(z,z))

: G = -(mu:^z)'x

: shat = ((y-mu):^z)'((y-mu):^z)*rows(x)/(rows(x)-cols(x))

: vb = luinv(G'w*G)*G'w*shat*w*G*luinv(G'w*G)

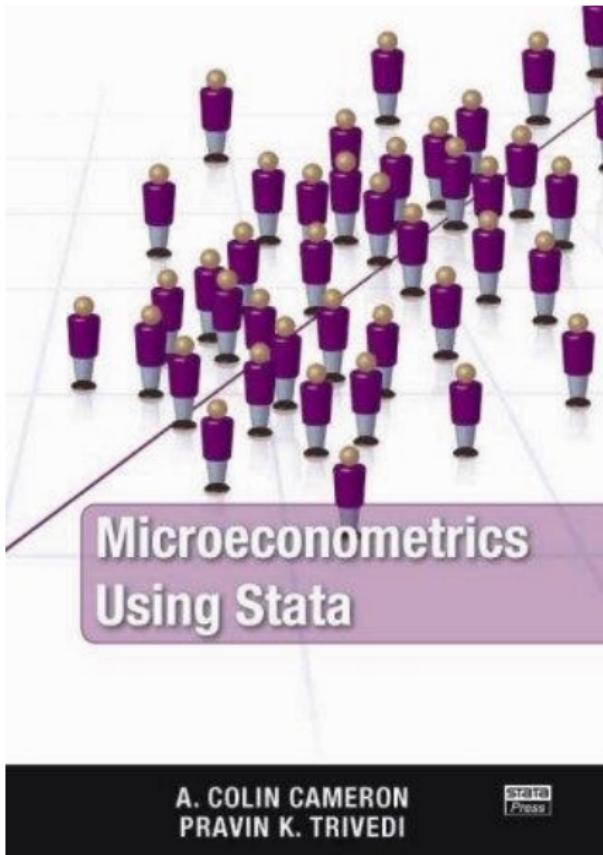
: seb = (sqrt(diagonal(vb)))'
```

```
: b \ seb
```

	1	2	3	4	5
1	1.340291853	1.072907529	.477817773	.0027832801	-.6832461817
2	1.559899278	.0763116698	.0690784466	.0021932119	1.350370916

```
: end
```

Coefficient of private of 0.799 becomes 1.340
and standard error of 0.109 becomes 1.559.



Book Outline

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3. Linear regression basics
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5. GLS regression
6. Linear instrumental variable regression
7. Quantile regression
8. Linear panel models: Basics
9. Linear panel models: Extensions

- 10.** Nonlinear regression methods
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 - 12.** Testing methods
 - 13.** Bootstrap methods
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 - 16.** Tobit and selection models
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