MAKING A MIRACLE

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This lecture surveys recent models of growth and trade in search of descriptions of technologies that are consistent with episodes of very rapid income growth. Emphasis is placed on the on-the-job accumulation of human capital: learning by doing. Possible connections between learning rates and international trade are discussed.

KEYWORDS: Growth, productivity, on-the-job training, learning.

1. INTRODUCTION

In 1960, the Philippines and South Korea had about the same standard of living, as measured by their per capita GDPs of about $640 U.S. 1975. The two countries were similar in many other respects. There were 28 million people in the Philippines and 25 million in Korea, with slightly over half of both populations of working age. Twenty seven percent of Filipinos lived in Manila, 28 percent of South Koreans in Seoul. In both countries, all boys of primary school age were in school, and almost all girls, but only about a quarter of secondary school age children were in school. Only 5 percent of Koreans in their early twenties were in college, as compared to 13 percent in the Philippines. Twenty six percent of Philippine GDP was generated in agriculture, and 28 percent in industry. In Korea, the comparable numbers were 37 and 20 percent. Ninety six percent of Philippine merchandise exports consisted of primary commodities and 4 percent of manufactured goods. In Korea, primary commodities made up 86 percent of exports, and manufactured goods 14 (of which 8 were textiles).

From 1960 to 1988, GDP per capita in the Philippines grew at about 1.8 percent per year, about the average for per capita incomes in the world as a whole. In Korea, over the same period, per capita income grew at 6.2 percent per year, a rate consistent with the doubling of living standards every 11 years. Korean incomes are now similar to Mexican, Portuguese, or Yugoslav, about three times incomes in the Philippines, and about one third of incomes in the United States.2

I do not think it is in any way an exaggeration to refer to this continuing transformation of Korean society as a miracle, or to apply this term to the very similar transformations that are occurring in Taiwan, Hong Kong, and Singapore. Never before have the lives of so many people (63 million in these four areas in 1980) undergone so rapid an improvement over so long a period, nor (with the tragic exception of Hong Kong) is there any sign that this progress is

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2 The figures in the first paragraph are taken from the 1984 World Development Report. The income and population figures in this paragraph and the next are from Summers and Heston (1991).
near its end. How did it happen? Why did it happen in Korea and Taiwan, and not in the Philippines?

Questions like these can be addressed at many levels. It is useful to begin simply by listing some of the features of these transformations in addition to their income growth rates. All of the East Asian miracle economies have become large scale exporters of manufactured goods of increasing sophistication. They have become highly urbanized (no problem for Singapore and Hong Kong!) and increasingly well-educated. They have high savings rates. They have pro-business governments, following differing mixes of laissez faire and mercantilist commercial policies. These facts—or at least some of them—must figure in any explanation of the growth miracles, but they are additions to the list of events we want to explain, not themselves explanations.

We want to be able to use these events to help in assessing economic policies that may affect growth rates in other countries. But simply advising a society to “follow the Korean model” is a little like advising an aspiring basketball player to “follow the Michael Jordan model.” To make use of someone else’s successful performance at any task, one needs to be able to break this performance down into its component parts so that one can see what each part contributes to the whole, which aspects of this performance are imitable and, of these, which are worth imitating. One needs, in short, a theory.

There has been a great deal of interesting new theoretical research on growth and development generally in the last few years, some of it explicitly directed at the Asian miracles and much more that seems to me clearly relevant. I will use this lecture to try and see what recent research offers toward an explanation for these events. My review will be sharply focused on neoclassical theories that view the growth miracles as productivity miracles. What happened over the last 30 years that enabled the typical Korean or overseas Chinese worker to produce 6 times the goods and services he could produce in 1960? Indeed, my viewpoint will be even narrower than the neoclassical theories on which I draw, since I intend to focus on issues of technology, with only cursory treatment of consumer preferences and the nature of product market competition. There is no doubt that the issue of who gets the rewards from innovation is a central one, and it is not one that can be resolved on the basis of technological considerations alone, so this narrow focus will necessarily restrict the conclusions I will be able to draw. But there is no point in trying to think through hard questions of industrial organization and general equilibrium without an adequate description of the relevant technology, so this seems to me the right place to start.

I will begin in Section 2, with a brief sketch of some recent theoretical developments and of the image of the world economy these developments offer. This image does not, as I see it, admit of anything one could call a miracle, but it will be useful in motivating my subsequent emphasis on the accumulation of human capital, and in particular on human capital accumulation on the job: learning by doing. In Section 3, I will review a piece of microeconomic evidence on learning and productivity, just to remind you how solid the evidence is and how promising, quantitatively, for the theory of growth. Yet establishing the
importance of learning by doing for productivity growth on a specific production process is very different from establishing its importance for an entire economy as a whole, or even for an entire sector. This connection is much more problematic than I once believed. But it has been made, in research by Nancy Stokey and Alwyn Young, and I will sketch the main technological implications of their work in Section 4. There is good reason to believe, I will argue, that something like this technology provided the means for the productivity miracles to occur. Section 5 discusses some of the issues involved in developing market equilibrium theories in which differential learning rates account for observed growth rate differences, and offers some speculations about the implications of such a theory for the development prospects of poor countries. Conclusions are in Section 6.

2. THEORETICAL BACKGROUND

There has been a rebirth of confidence—stimulated in large part by Romer's (1986) contribution—that explicit neoclassical growth models in the style of Solow (1956) can be adapted to fit the observed behavior of rich and poor economies alike, interacting in a world of international trade. I do not believe we can obtain a theory of economic miracles in a purely aggregative set-up in which every country produces the same, single good (and a rich country is just one that produces more of it) but such a framework will be useful in stating the problem and in narrowing the theoretical possibilities.

Consider, to begin with, a single economy that uses physical capital, \( k(t) \), and human capital, \( h(t) \), to produce a single good, \( y(t) \):

\[
(2.1) \quad y(t) = Ak(t)^a [uh(t)]^{1-a}.
\]

Here I multiply the human capital input by \( u \), the fraction of time people spend producing goods.\(^3\) The growth of physical capital depends on the savings rate \( s \):

\[
(2.2) \quad \frac{dk(t)}{dt} = sy(t),
\]

while the growth of human capital depends on the amount of quality-adjusted

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\(^3\) One of the referees for this paper found my use of the term “human capital” in this aggregate context idiosyncratic, and I agree that aggregate theorists tend to use terms like “technology” or “knowledge capital” for what I am here calling “human capital.” But the cost of having two terminologies for discussing the same thing, one used by microeconomists and another by macroeconomists, is that it makes it too easy for one group to forget that the other can be a source of relevant ideas and evidence.

It was the explicit theme of Schultz (1962) that the theory of human capital, then in its infancy, would prove central to the theory of economic growth, and Schultz included the stock of human capital accumulated on the job in his Table 1 (p. S6). His figures were based on estimates provided in Mincer (1962), whose estimation method “treats ‘learning from experience’ as an investment in the same sense as are the more obvious forms of on-the-job training, such as, say, apprenticeship programs” (p. S51). My usage in this paper is, I think, consistent with 30 years of practice in labor economics.
time devoted to its production:

\[
(2.3) \quad \frac{dh(t)}{dt} = \delta(1 - u)h(t).
\]

Taking the decision variables \( s \) and \( u \) as given, which I will do for this exposition, the model (2.1)–(2.3) is just a reinterpretation of Solow's original model of a single, closed economy, with the rate of technological change (the average Solow residual) equal to \( \mu = \delta(1 - \alpha)(1 - u) \) and the initial technology level equal to \( Ah(0)^{1-\alpha} \). In this system, the long run growth rate of both capital and production per worker is \( \delta(1 - u) \), the rate of human capital growth, and the ratio of physical to human capital converges to a constant. In the long run, the level of income is proportional to the economy's initial stock of human capital.\(^4\)

To analyze a world economy made up of countries like this one, one needs to be specific about the mobility of factors of production. A benchmark case that has the virtues of simplicity and, I think, a decent degree of realism is obtained by assuming that labor is completely immobile, while physical capital is perfectly mobile. That is, if there are \( n \) countries indexed by \( i \), assume that the world stock of physical capital, \( K = \sum_{i=1}^{n} k_i \), is allocated across countries so as to equate the marginal product in each country to a common world return, \( r \). Then if each country has the technology (2.1) with a common intercept \( A \), this world return is \( r = \alpha A(K/H)^{\alpha - 1} \), where \( H = \sum_{i} u_i h_i \) is the world supply of effective labor devoted to goods production. Net domestic product in each country is proportional to its effective workforce:

\[
(2.4) \quad y_i = A \left( \frac{K}{H} \right)^\alpha u_i h_i.
\]

If everyone has the same constant savings rate \( s \), the dynamics of this world economy are essentially the same as those of Solow's model. The world capital stock follows \( (dK/dt) = sAK^{\alpha}H^{1-\alpha} \), and the time path of \( H \) is obtained by summing (2.2) over countries, each multiplied by its own time allocation variable \( u_i \). The long run growth rate of physical capital and of every country's output is equal to the growth rate of human capital. Each country's income level will be proportional to its initial human capital, not only in the long run but all along the equilibrium path. The theory is thus consistent with the permanent maintenance of any degree of income inequality.

It would be hard to think of another theory as simple as this one that does a better job of fitting the postwar statistics in the back of the World Development Report. By reinterpreting Solow's technology variable as a country-specific stock

\(^4\) Of course, essentially the same economics can be obtained from a model in which consumer preferences are taken as given and savings and time allocation behavior are derived rather than assumed. See Uzawa (1965), Lucas (1988), and Caballé and Santos (1991). The particular model sketched in the text is simply one rather arbitrarily selected example from the large number of similarly motivated models that have recently been proposed. See, for example, Jones and Manuelli (1990), King and Rebelo (1990), and Becker, Murphy, and Tamura (1990).
of human capital, a model that predicts rapid convergence to common income levels is converted into one that is consistent with permanent income inequality. But the key assumption on which this prediction is based—that human capital accumulation in any one economy is independent of the level of human capital in other economies—conflicts with the evident fact that ideas developed in one place spread elsewhere, that there is one frontier of human knowledge, not one for each separate economy. Moreover, as Parente and Prescott (1991) observe, if the model above is realistically modified to permit each economy to be subject to shocks that have some independence across countries, the assumption that each economy undergoes sustained growth due to its own human capital growth only would imply ever-growing inequality within any subset of countries. Relative income levels would follow random-walk-like behavior. I do not see how this prediction can be reconciled with the postwar experience of, say, the OECD countries or the EEC. The countries of the world are tied together, economically and technologically, in a way that the model (2.1)–(2.3) does not capture.\footnote{An informative recent debate on income convergence has been stimulated by the exchange between Baumol (1986), De Long (1988), and Baumol and Wolff (1988). My statement in the text simply echoes the shared conclusion of these authors.}

One way to introduce some convergence into the model I have sketched, proposed and studied by Parente and Prescott (1991), is to modify the human capital accumulation technology (2.2) so as to permit any one country's rate of human capital growth to be influenced by the level of human capital elsewhere in the world. For example, let \( H(t) \) be the world effective labor variable defined above, and let \( Z(t) = H(t)/\sum \mu_i \) be the world average human capital level. Replace the human capital accumulation equation (2.2) with:\footnote{This external effect might better be captured through the human capital level of the most advanced countries, rather than the world average \( Z(t) \). But the use of the latter variable keeps the algebra simple, and I don't think the distinction is critical for any conclusions I wish to draw here.}

\[
(2.5) \quad \frac{dh(t)}{dt} = \delta(1-u)h(t)^{1-\theta}Z(t)^{\theta}.
\]

With this modification, the dynamics of the world stocks of physical and human capital are essentially unchanged, but now an economy with a human capital stock lower than the world average will grow faster than an above average economy. For example, if the time allocation is equal across countries, so that \( H(t) \) and \( Z(t) \) grow at the rate \( \delta(1-u) \), a country's relative human capital, \( z_i = h_i/Z \), follows

\[
(2.6) \quad \frac{d}{dt} z_i(t) = \delta(1-u)z(t)\left[z(t)^{-\theta} - 1\right].
\]

Evidently, \( z_i(t) \) converges to one, and from (2.4), this means that relative incomes converge to one at the same rate.

In the world as a whole in the postwar period, income dispersion across all countries appears to be increasing. But, of course, there are many reasons to believe that the assumption of free world trade that leads to (2.6) is a very bad
approximation for much of the world, and there are certainly differences across
countries in the incentives people have to accumulate both kinds of capital,
implying differences in savings rates and the allocation of time. Yet over subsets
of countries, or regions of countries, where factor and final goods mobility is
high (like the EEC or the 50 U.S. states) convergence can be observed.\footnote{7}

Barro and Sala-i-Martin (1992) obtain a regression estimate of an average
convergence rate of relative incomes, conditioned on variables that may be
interpreted as controlling for a country’s adherence to the above assumptions,
of slightly less than .02 (Table 3, p. 242). As they observe, if one interprets this
coefficient as reflecting differential rates of physical capital accumulation in a
world in which income differences reflect mainly differences in capital per
worker, this rate of convergence is much too low to be consistent with observed
capital shares. Alternatively, interpreting this figure as an estimate of
\((1/z)(dz/dt)\) in (2.6), their estimate implies \(\theta \delta (1 - u) = .02\). Since \(\delta (1 - u)\) is
the average rate of human capital growth, also about .02 in reality, this
interpretation yields an estimated \(\theta\) of unity, which from (2.5) would mean that
human capital accumulation in any country depends on local effort together
with worldwide knowledge, independent of the local human capital level. From
this viewpoint, the Barro-Sala-i-Martin estimate seems high.

All of this is by way of a prelude to thinking about growth miracles—about
deviations from average behavior. I have described a model of a world economy
—reasonably realistic in its description of average behavior of countries at
different income levels—in which everyone has the same savings rate and
allocates time in the same way. What are the prospects for using the same
theory to see how variations across economies in the parameters \(s\) and \(u\) can
induce variation in behavior of the magnitude we seek to explain? Here the
exercise begins to get hard.

The East Asian economies do indeed have high investment rates. The current
ratio of gross domestic investment to GDP in Korea is about .29, as compared
to average behavior of around .22. In Taiwan and Hong Kong, the investment
ratios are .21 and .24 respectively. In Singapore, it is a remarkable .47. In the
Philippines, for comparison it is .18.\footnote{8} In a world with the perfect capital mobility
used in my illustration above, these differences in investment rates would have
\textit{no} connection with savings rates: any country’s higher than average savings
would simply be invested abroad. Even with no international capital mobility, to
translate a given difference in savings rates into a differences in output growth
rates one must multiply by the return on capital (since

\[
\frac{\partial}{\partial s} \left( \frac{1}{y} \frac{dy}{dt} \right) = \frac{\partial}{\partial s} \left( \frac{1}{y} \frac{dy}{dt} \right) = \frac{\partial y}{\partial k},
\]

\footnote{7 See, for example, Ben-David (1991).}

\footnote{8 All the figures cited are for 1984. The ratio for Taiwan is from the 1987 Taiwan \textit{National Income.} The others are from the 1986 \textit{World Development Report}.}
from (2.2)). If the return on capital were ten percent, then, the Korea-Philippines investment rate difference of .11 can account for a difference of .011 in output growth rates, or about one percentage point. Even this effect is only transient, since in the long run differences in savings rates are level effects only.

Now applying the same rough calculation to the Singapore-Philippines investment rate difference of .29, one can account for a difference in output growth rates of nearly three percentage points (and more, if a higher and still defensible return on capital is used) which is close to the differentials I am calling "miraculous." Indeed, Young (1992) demonstrates that output growth in Singapore since the 1960's can be accounted for entirely by growth in conventionally measured capital and labor inputs, with nothing left over to be attributed to technological change. But Young's point, underscored by his parallel treatment of Singapore and Hong Kong, is the exceptional character of growth in Singapore, and not that the Asian miracles in general can be attributed to capital accumulation.

Growth accounting methods, applied country-by-country as in Young's study, can quantify the role of investment differentials in accounting for growth rate differences. In general, these differentials leave most measured output growth to be explained by other forces. This conclusion, which seems to me so clear, remains controversial. Correlations between investment ratios and growth rates, which tend to be positive, are frequently cited but do not settle anything. If growth is driven by rapid accumulation of human capital, one needs rapid growth in physical capital just to keep up: look at equation (2.4)! It may be that by excluding physical capital from the human capital accumulation equation (2.3) or (2.5) I have ruled out some interesting possibilities: One cannot accumulate skill as a computer programmer without a computer. Perhaps physical capital will assume a more important role when the technology for accumulating human capital is better understood, but if so, it will be at best a supporting part. Let us look elsewhere.

In the framework I am using, the other possible source of growth rate differentials is differential rates of human capital accumulation, stemming from differences in societies' time-allocation decisions. But human capital takes many forms and its accumulation occurs in many ways, so there are decisions in emphasis to be made here as well. The key choice, I think, is whether to stress human capital accumulation at school, or on the job.

If one interprets (2.3) or (2.5) as describing knowledge accumulation through schooling, these equations imply that doubling the fraction in school would double the human capital growth rate, adding only another .02 to the average rate of .02. And, of course, the linearity of (2.3) probably leads to an overstatement of the effect of so large a change. As I remarked in my introduction, the fast growing Asian economies are not, in general, better schooled than some of their slow growing neighbors. Emphasis on formal schooling, then, seems to involve the application of a modest multiplier to very slight differences in behavior, leading to the same discouraging conclusion for human capital that I arrived at in the case of physical capital.
This conclusion may seem an inappropriate inference from an oversimplified model, but I think it is in fact reinforced by thinking more seriously about the effects of schooling. Actual schooling decisions take place in a life-cycle context, with school preceding work and each individual deciding on the length of these two career phases. (This is a simplification, too, but a better one than thinking of a representative agent dividing his time in perpetuity.) Now in a steady state or balanced path of an economy in which everyone spends a fraction $1-u$ of his working life in school, workers with schooling level $1-u$ are retiring from the labor force at exactly the same rate as new workers with the same education level are entering. No matter what the value of $u$ is in such a steady state, all of this investment is replacement investment and there is no increase in the average skill level of the workforce. Since (2.3) is an hypothesis about net investment, one cannot then identify the variable $1-u$ with time spent in school. One is left with two choices. We can identify increases in average schooling levels with net human capital investment. Since schooling levels are increasing in virtually all societies today, this is a possibility worth developing, but it cannot be pursued within a steady state framework. This is an important and neglected respect in which neither advanced nor most backward economies can be viewed as moving along balanced growth paths.

Alternatively, we can think of a balanced path on which time spent in school is constant but the quality of schooling is improving due to increases in general knowledge. This possibility is analyzed in Stokey (1991a), from which the argument of the last paragraph is taken. In this paper, the rate of expansion of knowledge is taken to be an external effect of the time spent in school, the hypothesis that transforms a level effect into the needed growth effect. But this hypothesis does not salvage the multiplier arguments I applied above, unless one is willing to assume that increases in general knowledge accrue equally from time spent in primary schools and universities. To quantify a model like Stokey's, one would need a much sharper empirical identification of the set of activities that lead to new knowledge—to net investment in a society's human capital—than is provided by any aggregate index of total schooling time. This would be a most interesting avenue to explore but I am not prepared to do so here, so I will end this digression and move on.

Human capital accumulation also occurs at work, as we know from the fact the experienced workers and managers earn more than inexperienced ones. This aspect of human capital accumulation—on the job training—could also be (and has been) modeled as a time-allocation decision. Alternatively, in a multiple good world, one could think of on the job accumulation—learning by doing—as associated with the type of process one is engaged in. That is, one might think of some activities as carrying with them a high rate of skill acquisition and others, routine or traditional ones, as associated with a low rate. If so, the mix of goods a society produces will affect its overall rate of human capital accumulation and growth. For understanding diversity, I think this route has promise: The variation across societies, or at least those engaged in international trade, in the mix of goods produced is enormous. In this section, I
have tried to motivate a focus on this source of diversity by a process of elimination: Neither physical capital accumulation nor human capital accumulation through schooling seems to have much potential, at least within the framework I have adopted. In this next section, I turn to much more direct, microeconomic evidence on the same point.

3. THE LIBERTY SHIP MIRACLE

In Lucas (1988) I used a multi-good model, adapted from Krugman (1987), in which different goods were associated with different learning rates to capture the idea that the choice of which goods to produce can be viewed as an implicit choice of a human capital accumulation rate. In a world of open economies, comparative advantage—previously accumulated, good-specific human capital holdings—will determine who produces what, and the mix of goods that this process assigns to a particular economy will determine its rates of human capital growth. This kind of formulation has been taken in interesting directions by Boldrin and Scheinkman (1988) and Matsuyama (1992). It is attractive, for present purposes, because there are such wide differences in product mix across countries and because the fast growing Asian economies have undergone such dramatic changes in the goods they produce.

But the hypothesis that different goods are associated with permanently different learning potentials conflicts sharply with available evidence in two respects. First, examination of growth in total factor productivity (Solow residuals) across both industries and time (as conducted, for example, by Harberger (1990), shows no decade-to-decade stability in the high productivity growth industries. Lumber and wood products can rank 14th in the 1950’s, first in the 1960’s, and disappear from the list of leaders altogether in the 1970’s.9 Second, evidence we have on learning on narrowly defined product lines invariably shows high initial learning rates, declining over time as production cumulates. These two kinds of evidence reinforce each other, and seem decisive against the formulation Krugman proposed. These observations have led Stokey (1988) and Young (1991a) to a very different formulation, one that is much more tightly grounded in microeconomic evidence. I will review this formulation in Section 4, but before doing so I want to reinforce the motivation with a reminder of just how impressive the evidence on the productivity effects of learning by doing can be.

The best evidence I know of that bears on on-the-job productivity change in a single, large scale production process, was utilized in studies by Allan D. Searle (1945) and Leonard A. Rapping (1965). Both studies used data on the production of a single type of cargo vessel—the Liberty Ship—in 14 U.S. shipyards during World War II. From December, 1941, through December, 1944, these yards produced a total of 2458 Liberty Ships, all to the same standardized design. For several individual yards, Searle plotted man-hours per vessel against

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9 Harberger (1990), Table 3.
number of vessels completed to date in that yard on log-log paper. His results for two yards are reproduced here as Figure 1. Average results over ten yards are given in Figure 2, along with results for three other vessel types. For Liberty Ships, "the reductions in manhours per ship with each doubling of cumulative output ranged from 12 to 24 percent."\textsuperscript{10}

Stimulated in part by Kenneth Arrow’s (1961) theoretical suggestion that learning-by-doing might serve as the key factor in growth for an economy as a whole, Rapping incorporated Searle’s and other evidence within a neoclassical production framework. He pooled the data for all yards and estimated a Cobb-Douglas production function, controlling for changes in capital per yard, with cumulated yard (not industry) production as an added regressor. He obtained estimates of the learning effect, comparable to Searle’s, ranging from 11 to 29 percent. He also showed that the inclusion of calendar time added nothing (the trend came out slightly negative!) to these results.

I do not think there is anything unique to shipbuilding in the findings that Searle and Rapping obtained. The Boston Consulting Group (1972) has obtained fairly clean learning curves, with slopes similar to those estimated by Searle and Rapping, for a variety of industries, and other researchers have done

\textsuperscript{10}Searle (1945), p. 1144.
Figure 2.—Unit man-hour requirements for selected shipbuilding programs. Vessels delivered December 1941–December 1944.
so as well. What is unique about the Liberty ship data is that the ships were built according to exactly the same blueprints over a period of several years and that data were available yard by yard. Figure 2, which gives Searle’s learning curve for the industry as a whole, is not nearly as sharp as the curves in Figure 1 for individual yards, presumably because industry expansion is a mix of increased production by existing yards and the entry of new, inexperienced yards. Production data even from narrowly defined industries mask continual model and other product mix changes over time, which makes it difficult to use them to identify even strong learning effects. What is exceptional about the Liberty ship evidence, I think, is the cleanness of the experiment, not the behavior it documents so beautifully.

Quantitatively, these results are interesting to an economist looking for possible sources of miracles. For the three year period covered by Rapping’s study, industry output per manhour increased at a 40 percent annual rate! There is also considerable ambiguity about what this evidence means. Is it the individual worker who is doing the learning? The managers? The organization as a whole? Are the skills being learned specific to the production process on which the learning takes place, or more general? Does learning accrue solely to the individual worker, manager, or organization that does the producing, or is some of it readily appropriable by outside observers? These are questions that the theory of growth needs to address, but I will pass over them here.

A more urgent question, I think, is whether the kind of behavior Rapping and Searle documented, for one product line for one brief period, can be linked to productivity growth for an entire economy over periods of thirty or forty years. This is the topic of the next section.

4. LEARNING MODELS: TECHNOLOGY

In order to examine the possible connection between evidence of learning on individual product lines and productivity growth in an economy as a whole, consider the labor-only technology:

\[ (4.1) \quad x(t) = kn(t)z(t)^\alpha, \]

where \( x(t) \) is the rate of production of a good, \( k \) is a productivity parameter that depends on the units in which labor input and output are measured, \( n(t) \) is employment, and \( z(t) \) represents cumulative experience in the production of this good. Cumulative experience is in turn defined by the differential equation:

\[ (4.2) \quad \frac{dz(t)}{dt} = n(t)z(t)^\alpha, \]

and the initial value \( z(t_0) \), assumed to be greater than or equal to one, of the experience variable on the date \( t_0 \) when production was begun. The general
solution to (4.2) is
\[
(4.3) \quad z(t) = \left[ (z(t_0))^{1-\alpha} + (1 - \alpha) \int_{t_0}^{t} n(u) \, du \right]^{1/(1-\alpha}).
\]

The implications of this model for the dynamics of production of a single good are familiar enough. Suppose, to take the simplest case, that employment is constant at $\bar{n}$ over time. Then (4.1) and (4.3) imply that production follows
\[
x(t) = k\bar{n} \left[ z(t_0)^{1-\alpha} + (1 - \alpha) \bar{n}(t - t_0)^{1/(1-\alpha)} \right].
\]
Production grows without bound, and the rate of productivity growth declines monotonically from $a\bar{n}(z(t_0))^{\alpha-1}$ to zero. For any initial productivity level $z(t_0) > 1$ and any employment level (or path) productivity at date $t$ is an increasing function of the learning rate $\alpha$.

Notice that the technology (4.2) implies a scale effect: a link between the level of employment and the rate of growth of productivity. This carries the unwelcome implication that a country like India should have an enormous growth advantage over a small country like Singapore. This is a feature of any learning by doing theory, but I agree with Matsuyama (1992) that if one is thinking about an entire economy or sizeable sector of an economy, it is a nuisance implication that we want to dispose of.\footnote{Backus, Kehoe, and Keohoe (1991) is an empirical examination of scale effects on growth rates, formulated in a variety of ways. They find some evidence of such effects in manufacturing, and none for economies as a whole.} Matsuyama proposes thinking of a population as containing a fixed fraction of entrepreneurs, and of a technology that requires that each enterprise be headed by one of them. Then doubling the population means doubling the number of enterprises that are subject to the learning technology, keeping the size of each fixed, and has no growth effects. Insofar as learning effects are partly external to the firm, as I think they are, this device doesn’t quite work, and one needs to think of some other limitation on scale—city size, say. I will simply ignore these scale economies in what follows, assuming that some explanation along the lines of Matsuyama’s will be discovered to rationalize this neglect.

With the technology (4.1)–(4.3), one can obviously obtain miraculous rates of productivity growth by shifting a large amount of labor onto a single, new product line. Provided that $\bar{n}(t - t_0)$ is large relative to initial experience (which is the way most people interpret statistical learning curves), the rate of productivity growth $t$ years after production is initiated is approximately $\alpha / ((1 - \alpha)t)$. Using the value $\alpha = 0.2$ estimated by Rapping and Searle, productivity growth one year after a product is introduced is $\alpha / (1 - \alpha) = 0.25$. After two years, the growth rate is reduced by half to 0.125, and so on. A growth miracle sustained for a period of decades clearly must thus involve the continual introduction of new goods, not merely continued learning on a fixed set of goods. Even if new goods are introduced, a shift of workers from old goods with low learning rates to new goods with high rates involves an initial drop in productivity: people are
better at familiar activities than they are at novel ones. It is not even clear how these factors balance out.

To pursue this question, I follow Stokey (1988) and consider an economy in which a variety of goods, indexed by \( s \), is produced, where a higher index \( s \) means a better good. In Stokey (1988) and, in different ways in Young (1991a) and Grossman and Helpman (1991b), specific assumptions on consumer preferences or the technology give a precise meaning to the sense in which one good is better than another. For my immediate objectives, it will be adequate to consider a small, open economy and to use an assumed schedule \( p(s, t) = e^{\mu s} \) of world prices to summarize the quality of goods: a better good means a good with a higher price on world markets. Assume that the economy progresses by introducing better quality (higher \( s \)) goods into production over time, and let \( S(t) \) be the index of the good that is first produced at date \( t \). (I will also use \( \tau(s) \), where \( \tau \) is the inverse function of the increasing function \( S \), to denote the date on which good \( s \) is first produced.) Then if \( x(s, t) \) is production of good \( s \) at date \( t \), the value of the economy's total production is

\[
y(t) = \int_{0}^{S(t)} e^{\mu s} x(s, t) \, ds.
\]

Let \( n(s, t) \) be employment on good \( s \) at \( t \), and \( z(s, t) \) be cumulated experience. Then if learning proceeds independently, good by good, (4.1) and (4.3) imply

\[
x(s, t) = kn(s, t) \left[ \left( z(s, \tau(s)) \right)^{1-\alpha} + (1 - \alpha) \int_{\tau(s)}^{t} n(s, u) \, du \right]^{\alpha/(1-\alpha)}.
\]

Equations (4.4) and (4.5) together describe the implications for total production of a given way of allocating labor across product lines through time.

Consider the following specific labor allocation. Let the rate of new product introduction be a constant \( \lambda \), so that \( S(t) = \lambda t \) and \( \tau(s) = s/\lambda \). Let \( \varphi \) be a density function with cdf \( \Phi \), and suppose that for all \( s \in (0, \lambda t] \), \( n(s, t) = \varphi(t - s/\lambda) \) (that \( \varphi(t - s/\lambda) \) workers are assigned to produce the goods of age \( t - s/\lambda \)) and that the remaining \( 1 - \Phi(t) \) workers produce a good 0 on which no learning occurs. Assume that initial productivity is the same for all goods, at the level \( z(s/\lambda, s) = \xi \geq 1 \). Under these assumptions, (4.4) and (4.5) imply that the value of total production is

\[
y(t) = 1 - \Phi(t) + k \lambda e^{\mu \lambda t} \int_{0}^{t} e^{-\mu \lambda u} \varphi(u) \left[ \xi^{1-\alpha} + (1 - \alpha) \Phi(u) \right]^{\alpha/(1-\alpha)} \, du.
\]

The asymptotic growth rate for this economy is evidently \( \mu \lambda \). This rate does not depend either on the learning parameter \( \alpha \) or on the distribution \( \varphi \) of the workforce over goods of different vintages. Changes in either of these factors are simply level effects. To obtain sustained growth at all in this framework, it is necessary to assume that better goods become producible at some exogenously given rate \( \lambda \), which then along with the quality gradient \( \mu \) dictates the long run growth rate of the system, independent of learning behavior.
Though the production of new goods is continuously initiated in this example, the rate at which this occurs through time is fixed. In Stokey (1988) this rate is made endogenous through the assumption that the experience accumulated in producing good $s$ reduces the cost of producing good $s' > s$. (It may reduce the cost of producing $s' < s$, too, but the spillover effect is assumed to be loaded in the direction of improving productivity on the more advanced good.) As a specific instance of Stokey’s hypothesis, very close to that proposed by Young (1991a), let us modify the last example by postulating that the initial value $z(s, \tau(s))$ in the learning curve (4.3) depends on the experience that has been accumulated on less advanced goods. Suppose that an economy at some fixed date $t$ has experienced summarized by $z(s, t)$ for $s < S(t)$, but has yet to produce any good with index above $S(t)$. Assume that if production of a good $s \geq S(t)$ is initiated at $t$ (if $\tau(s) = t$) then its initial $z$-value is proportional to an average of the economy’s experience on previously produced goods:

\[
(4.7) \quad z(s, \tau(s)) = \theta \delta \int_0^s e^{-\delta(s-u)}z(u, \tau(s)) \, du.
\]

Equation (4.7) expresses the initial productivity on good $s$ as an average of experience on lower quality goods. Equivalently, we can express the initial productivity on the good introduced at $t$, good $S(t)$, as an average of experience on goods introduced earlier:

\[
(4.8) \quad z(S(t), t) = \theta \delta \int_0^t e^{-\delta \left[S(t)-S(t-v)\right]}z(S(t-v), t)S'(t-v) \, dv,
\]

integrating over ages $v$ instead of goods $s$.

Assume, next, that production on a new good is initiated whenever the expressions (4.7) and (4.8) reach a trigger value $\xi \geq 1$, taken as a given constant. Under this assumption, the left side of (4.8) is replaced with this constant $\xi$, implying that the function $S(t)$ whose derivative is the rate at which new goods are introduced must satisfy

\[
(4.9) \quad \xi = \theta \delta \int_0^t e^{-\delta \left[S(t)-S(t-v)\right]}z(S(t-v), t)S'(t-v) \, dv.
\]

As in the previous example, we continue to assume that the allocation of employment at any date is described by a density $\varphi$ and cdf $\Phi$, where $\Phi(u)$ is the fraction of people employed producing goods that were introduced less than $u$ years earlier. In the present case, each good has the initial productivity level $\xi$, so inserting the solution (4.3) for $z(S(t-v), t)$ with this initial value into (4.9) yields a single equation in the function $S(t)$. For large values of $t$, the solution $S(t)$ to this equation will behave like $S(t) = \lambda t$, where the constant $\lambda$ satisfies

\[
(4.10) \quad \xi = \theta \delta \lambda \int_0^\infty e^{-\delta \lambda v} \left[\xi^{1-\alpha} + (1-\alpha)\Phi(v)\right]^{1/(1-\alpha)} \, dv.
\]

The right side of (4.10) is just an average of the positive, increasing function $\theta[\xi^{1-\alpha} + (1-\alpha)\Phi(v)]^{1/(1-\alpha)}$, taken with respect to an exponential distribution with parameter $\delta \lambda$. Hence it is a positive, decreasing function of $\delta \lambda$, tending
toward the value $\theta \xi$ as $\delta \lambda \to \infty$ and toward the value $\theta [\xi^{1-\alpha} + 1 - \alpha]^{1/(1-\alpha)}$ as $\delta \lambda \to 0$. (If the latter expression is less than $\xi$ at $\lambda = 0$, then the economy does not accumulate relevant experience fast enough to introduce new goods in the steady state.) For fixed $\delta \lambda$, the right side of (4.10) is an increasing function of $\theta$, $\alpha$, and $k$, and it also increases as the distribution of labor $\varphi(\nu)$ becomes more concentrated on lower values of $\nu$ (on newer goods). Hence if a positive solution $\lambda$ exists, it is inversely proportional to the decay rate of spillover experience, an increasing function of the spillover parameter $\theta$ and the learning rate $\alpha$, and increases as employment is more heavily concentrated on goods that are closer to the economy's production frontier.

The formula (4.6) for the value of total production continues to hold in this second example, and the economy's long run growth rate is $\lambda \mu$, as before. But under this second, spillover, technology, economies that distribute workers across goods of different ages in different ways will grow at different rates. Of course, this conclusion is not based purely on technological considerations: The value $\xi$ of initial productivity that is assumed to trigger the initiation of production of a new good is of central importance, and needs an economic rationale.

One might view the spillover technologies of Stokey and Young as reconciling the Krugman hypothesis of a manufacturing sector with a constant rate of productivity growth, based on learning, with the fact that learning rates on individual production processes decline over time to zero. For example, one could interpret either of the examples in this section as describing a sector of an economy with a positive asymptotic rate of productivity growth. On this view, the contribution of Stokey and Young is to break down an assumed sectoral learning rate into its components, $\alpha$, $\theta$, and $\delta$ (in my notation), and to relate this rate to the way workers are distributed over goods of different vintages.

This interpretation seems fine to me as long as one is discussing the consequences of a given workforce distribution, but if one has in mind applying the theory of comparative advantage to determining the way workers in each country are allocated to the production of different goods it ceases to make sense. In Krugman's theory (as in Lucas (1988)) it is a sector as a whole that either has or does not have a comparative advantage. In a sectoral interpretation of Stokey and Young's theories, each sector consists of many goods and comparative advantage must be determined good by good. No country can be expected to have a comparative advantage in manufacturing in general, or even in crude aggregates like Chemicals and Allied Products or Printing and Publishing. Comparative advantage will be associated with categories, like acetylene or paperback editions of English poetry, that are invisible even in the finest industrial statistics. As we shall see in the next section, this feature—besides being a step towards greater realism—leads to an entirely different view of trade and growth than is implied by the Krugman technology, the superficial similarity of the two notwithstanding.

The main attraction of a learning spillover technology such as that described in the second example of this section is that it offers the potential of accounting
for the great difference in productivity growth rates that are observed among low and middle income economies. Of course, little is known about the crucial spillover parameters $\delta$ and $\theta$—on which the learning curve evidence described in Section 3 provides no information—but surely an essential first step is to find a formulation that is capable, under some parameter values, of generating the behavior we are trying to explain.

5. LEARNING AND MARKET EQUILIBRIUM

The objective of the last section was to set down on paper a technology that is consistent with a growth miracle, which is to say, consistent with wide differences in productivity growth among similarly endowed economies. This has been done, following Stokey and Young, in a way that I think is consistent with the main features of the East Asian miracles, all of which have involved sustained movement of the workforce from less to more sophisticated products. A fast growing economy or sector under this technology is one that succeeds in concentrating its workforce on goods that are near its own quality frontier, and thus in accumulating human capital rapidly through the high learning rates associated with new activities and through the spillover of this experience to the production of still newer goods. These hypotheses are consistent with commonly known facts, and have testable implications for many more. As yet, however, I have said nothing about the economics that determine the mix of production activities in which an economy or sector of an economy in fact engages.

The papers of Stokey (1988), (1991b) and Young (1991a) develop models of market equilibrium with learning technologies under the assumption the effects of learning are external—that all human capital is a public good. In this case, labor is simply allocated to the use with the highest current return, independent of learning rates. With the constant returns technology these authors assume, the competitive equilibrium is Ricardian and straightforward to calculate. This is the simplest case, so I will begin with it too.

In such a setting, Stokey (1991b), studies north-south trade, where “north” means relatively well-endowed with human capital. Under specific assumptions about consumer preferences for goods of different qualities, she obtains a unique world equilibrium in which the south produces an interval of low quality goods, the north produces an interval of high quality goods, and there is an intermediate range of goods that are produced in neither place. With free trade (as opposed to autarky) learning-by-doing is depressed in the poor country, which now imports high-quality goods from the rich country rather than attempting to produce them at home. One can see that with dynamics as assumed in Stokey (1988), both countries will enjoy growth but the poor country will remain forever poorer.

A similar equilibrium is characterized in Young (1991a), using a parameterization of preferences and the learning technology that permits the explicit calculation of the north-south equilibrium, including a full description of the equilibrium dynamics. There are many possible equilibrium evolutions of his
north-south system, depending on the populations of the two regions and on their relative human capital holdings at the time trade is initiated. As in Stokey’s (1991b) analysis, the advanced country produces high quality goods and the poor country produces low quality goods. Free trade slows learning and growth in the poor country and speeds it in the rich one. In Young’s framework, there are equilibria in which the poor catch up to the rich, but only when their larger population lets them enjoy greater scale economies. Young does not emphasize this possibility and, as I have said earlier, I do not wish to either.

The equilibria of Stokey and Young, then, involve sustained growth of both rich and poor, at possibly different rates, and the continuous shifting of production of goods introduced in the north to the lower wage south. Initial comparative advantage is not permanent, as in Krugman’s formulation, since a rich country’s experience in producing any given good will eventually be offset by the fact that the good can be produced more cheaply in a less experienced but lower wage environment. Yet there are no growth miracles in these theories. Though these equilibria could readily be modified to include cross-country external effects, and hence catching up (for reasons unrelated to economies of scale), as I have done with the Solow model, there would be nothing one would wish to call miraculous about this process.

In the models of Stokey and Young, all human capital benefits are assumed to be external. The learning and growth that occurs is always, in a sense, accidental. Other models contain aspects of privately held knowledge, so that individual agents face the capital-theoretic problem of balancing current returns against the future benefits of learning of some kind. Matsuyama (1991) studies a two-sector system in which workers compare the present value of earnings in a traditional sector to the value of earnings in a manufacturing sector in which production is subject to external increasing returns. Young (1991b) augments learning with a research activity that yields patentable new products. Grossman and Helpman (1991a) postulate two R and D activities—innovation, done only in advanced economies, and imitation, done by poor economies too—with lags that let the discoverer or successful low-cost imitator enjoy a period of supernormal profits in a Bertrand-type equilibrium. Whether one calls the decision problems that arise in these analyses occupational choice, or research and development, or learning, all involve a decision on the allocation of time-at-work that involves balancing current returns against the benefits of increased future earnings, and all have a similar capital-theoretic structure.

Dropping the assumption that learning has external effects only is certainly a step toward realism, one that raises many interesting theoretical possibilities yet to be explored. It is thus only conjecture, but I would guess that the main features of the equilibria that have been worked out by Stokey and Young will turn out to stand up very well under different assumptions about the ownership, if I can use that term, of human capital. A learning spillover technology gives those who operate near the current goods frontier a definite advantage in moving beyond it. This advantage is decisive when decisions are taken myopically; I do not see why it should disappear when some of the returns from doing
so are internalized and workers and firms look to the future in their individual decision problems.

In short, available general equilibrium models of north-south trade do not predict miraculous economic growth for the poor countries taken as a group, nor do I see any reason to expect that the equilibria of more elaborate theories will have this feature. This is a disappointment, perhaps, but it does not seem to me to be a deficiency of these models. These are theories designed to capture the main interactions between the advanced economies taken as a group and the backward economies as a whole, within a two-country world equilibrium framework. Since it is a fact that the poor are either not gaining on the rich or are gaining only very slowly, one wants a theory that does not predict otherwise.

A successful theory of economic miracles should, I think, offer the possibility of rapid growth episodes, but should not imply their occurrence as a simple consequence of relative backwardness. It should be as consistent with the Philippine experience as with the Korean. For the purpose of exploring these possibilities, the conventions of small, open economy trade theory are more suitable (as well as simpler to apply) than those of the theory of a closed, two-country system. If the technology available to individual agents facing world prices has constant returns, then anything is possible. Some allocations will yield high external benefits and growth in production and wages; others will not. There will be a large number of possibilities, with individual agents in equilibrium indifferent between courses of action that have very different aggregative consequences. Theoretically, one can shut off some of these possibilities by introducing diminishing returns in the right places, but I am not sure that these multiplicities should be viewed as theoretical defects, to be patched up. If our objective is to understand a world in which similarly situated economies follow very different paths, these theoretical features are advantageous. A constant returns (at the level of individual producing units) learning spillover technology is equally consistent with fast and slow growth. If our task is to understand diversity, this is an essential feature, not a deficiency.

A second attraction of the learning spillover technology is that it is consistent with the strong connection we observe between rapid productivity growth and trade or openness. Consider two small economies facing the same world prices and similarly endowed, like Korea and the Philippines in 1960. Suppose that Korea somehow shifts its workforce onto the production of goods not formerly produced there, and continues to do so, while the Philippines continues to produce its traditional goods. Then according to the learning spillover theory, Korean production will grow more rapidly. But in 1960, Korean and Philippine incomes were about the same, so the mix of goods their consumers demanded was about the same. For this scenario to be possible, Korea needed to open up a large difference between the mix of goods produced and the mix consumed, a difference that could widen over time. Thus a large volume of trade is essential to a learning-based growth episode.

One can use the same reasoning to see why import-substitution policies fail, despite what can initially appear to be success in stimulating growth. Consider
an economy that exports, say, agricultural products and imports most manufactured goods. If this economy shifts toward autarky through tariff and other barriers, its workforce will shift to formerly imported goods and rapid learning will occur. But this is a one-time stimulus to productivity, and thereafter the mix of goods produced in this closed system can change only slowly, as the consumption mix changes. Note that this argument has to do only with the pace of change in an economy's production mix and does not involve scale, though it can obviously be reinforced by scale economies.

I do not intend these conjectures about the implications of a learning spillover technology for small countries facing given world prices to be a substitute for the actual construction of such a theory. To do this, one would need to take a realistic position on these issues touched on in my discussion of Rapping's and Searle's evidence. What is the nature of the human capital accumulation decision problems faced by workers, capitalists, and managers? What are the external consequences of the decisions they take? The papers cited here consider a variety of possible assumptions on these economic issues, but it must be said that little is known, and without such knowledge there is little we can say about the way policies that affect incentives can be expected to influence economic growth.

6. CONCLUSIONS

I began by asking what current economic theory has to say about the growth miracles of East Asia. The recent literature on which I have drawn to answer this question is fragmentary, and my survey of it more fragmentary still. Even so, the image of the growth process and the role of these remarkable economies within this process that emerges is, I think, surprisingly sharp, certainly compared to what could have been said on this subject ten years ago. I will conclude by summarizing it.

The main engine of growth is the accumulation of human capital—of knowledge—and the main source of differences in living standards among nations is differences in human capital. Physical capital accumulation plays an essential but decidedly subsidiary role. Human capital accumulation takes place in schools, in research organizations, and in the course of producing goods and engaging in trade. Little is known about the relative importance of these different modes of accumulation, but for understanding periods of very rapid growth in a single economy, learning on the job seems to be by far the most central. For such learning to occur on a sustained basis, it is necessary that workers and managers continue to take on tasks that are new to them, to continue to move up what Grossman and Helpman call the “quality ladder.” For this to be done on a large scale, the economy must be a large scale exporter.

This picture has the virtue of being consistent with the recent experience of both the Philippines and Korea. It would be equally consistent with post-1960 history with the roles of these two economies switched. It is a picture that is consistent with any individual small economy following the East Asian example,
producing a very different mix of goods from the mix it consumes. It does not appear to be consistent with the third world as a whole beginning to grow at East Asian rates: There is a zero-sum aspect, with inevitable mercantilist overtones, to productivity growth fueled by learning by doing.

Can these two paragraphs be viewed as a summary of things that are known about economic growth? After all, they are simply a sketch of some of the properties of mathematical models, purely fictional worlds, that certain economists have invented. How does one acquire knowledge about reality by working in one’s office with pen and paper? There is more to it, of course: Some of the numbers I have cited are products of decades-long research projects, and all of the models I have reviewed have sharp implications that could be, and have not been, compared to observation. Even so, I think this inventive, model-building process we are engaged in is an essential one, and I cannot imagine how we could possibly organize and make use of the mass of data available to us without it. If we understand the process of economic growth—or of anything else—we ought to be capable of demonstrating this knowledge by creating it in these pen and paper (and computer-equipped) laboratories of ours. If we know what an economic miracle is, we ought to be able to make one.

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REFERENCES


KRUGMAN, PAUL R. (1987): “The Narrow Moving Band, the Dutch Disease, and the Consequences
of Mrs. Thatcher: Notes on Trade in the Presence of Scale Economies,” Journal of Development
Economics, 27, 41–55.
Economics, 22, 3–42.
Political Economy, 70, S50–S79.
RAPING, LEONARD A. (1965): “Learning and World War II Production Functions,” Review of
Economics and Statistics, 47, 81–86.
ROMER, PAUL (1986): “Increasing Returns and Long-Run Growth,” Journal of Political Economy,
94, 1002–1037.
70, S1–S8.
SEARLE, ALLAN D. (1945): “Productivity Changes in Selected Wartime Shipbuilding Programs,”
of Economics, 70, 65–94.
Political Economy, 96, 701–717.
106, 587–616.
SUMMERS, ROBERT, and ALAN HESTON (1991): “The Penn World Table (Mark 5): An Expanded Set
UYAWA, HIROYUKI (1965): “Optimum Technical Change in an Aggregative Model of Economic
——— (1992): “A Tale of Two Cities: Factor Accumulation and Technical Change in Hong Kong