Appendix to Chapter 9

INTEREST RATES AND TIME PREFERENCE

Introduction

Growth may have been slow before 1700, but dramatic change did occur in the capital market. The rate of return fell sharply in this period from more than 10% to less than 3% in the case of the Netherlands by the late seventeenth century. Since the rate of return on capital is one of the fundamental prices in any economy, this potentially had major impacts. Here we ask what was the rate of return on capital in pre-industrial society and why was it so high?

Why the Rate of Return Matters

The rate of return on capital is one of the most important prices in any economy, because it determines how much future benefits and costs are weighed relative to current benefits and costs. At a 3% rate of return, for example, $100 invested now will be worth the amounts shown in table 1 in ten, twenty, thirty, forty and fifty years. If you were prepared to wait thirty years you could
### TABLE 1: RETURNS FROM WAITING AT DIFFERENT RATES OF RETURN

<table>
<thead>
<tr>
<th>No. of years</th>
<th>Interest Rate 3% ($)</th>
<th>Interest Rate 5% ($)</th>
<th>Interest Rate 10% ($)</th>
<th>Interest Rate 12% ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>10</td>
<td>134</td>
<td>163</td>
<td>259</td>
<td>311</td>
</tr>
<tr>
<td>20</td>
<td>181</td>
<td>265</td>
<td>673</td>
<td>965</td>
</tr>
<tr>
<td>30</td>
<td>243</td>
<td>432</td>
<td>1,745</td>
<td>2,996</td>
</tr>
<tr>
<td>40</td>
<td>326</td>
<td>704</td>
<td>4,526</td>
<td>9,305</td>
</tr>
<tr>
<td>50</td>
<td>438</td>
<td>1,147</td>
<td>11,739</td>
<td>28,900</td>
</tr>
</tbody>
</table>
consume 2.4 times as much as you can consume from $100 now at a 3% real interest rate. At a 10% interest rate you could consume 30 times as much in 30 years. As a converse of this table it implies that with a 10% rate of return on capital $1 received twenty years from now is worth only 15¢, and $1 received thirty years from now 6¢. Thus as the rate of return rises future costs and benefits become less and less important in guiding peoples’ actions now. The future matters less and less in the everyday decisions of people.

To illustrate this consider the following examples of land leases. When London was expanding greatly in the nineteenth century much agricultural land was sold for building housing. Often land owned by noble families in England could not be sold outright, because ancestors of the current owners had signed covenants restricting sale of the land (in order to ensure that the assets of the family were not squandered by a spendthrift heir). Thus much of London was built on land which was only leased to the homeowners for 99 years. When many of these leases expired in the 1960s and 1970s some of the most valuable real estate in England reverted to these noble families. Thus the Duke of Westminster is the richest person in England. What was the cost in 1860 of being able to sell only a 99 year lease on land as opposed to selling it outright?

The capital value of a piece of land which has an annual rent of $ per year will be

$$ P = \frac{v}{1+r} + \frac{v}{(1+r)^2} + \frac{v}{(1+r)^3} + \ldots = \frac{v}{r} $$
Thus if the land rents for £100 per year, and the interest rate is 5%, then the value of the land will be £2,000. The value of a lease for 99 years will be

\[
P_{99} = \frac{v}{(1+r)} + \frac{v}{(1+r)^2} + \frac{v}{(1+r)^3} + \ldots + \frac{v}{(1+r)^{99}}
\]

\[
= \frac{v}{r} \left[ \frac{v}{(1+r)^{100}} + \frac{v}{(1+r)^{101}} + \frac{v}{(1+r)^{102}} + \ldots \right]
\]

\[
= \frac{v}{r} \left[ \frac{1}{(1+r)^99} \left( \frac{1}{(1+r)^1} + \frac{v}{(1+r)^2} + \frac{v}{(1+r)^3} + \ldots \right) \right]
\]

\[
= \frac{v}{r} \left[ 1 - \frac{1}{(1+r)^99} \right]
\]

\[
P_{99} = P \left[ 1 - \frac{1}{(1+r)^99} \right]
\]

At a 5% interest rate 1/(1+r)^99 is equal to .008. Thus if the land would sell outright for £2,000, it would lease for 99 years for £1,984, £6 less than for outright sale. Thus the covenants that limited land sales by noble families in Britain in the nineteenth century cost the people at the time almost nothing in terms of income. But it preserved for future generations of these families a huge wealth in land. The future rents from years 100 to the end of time have a tiny effect on the current value of the land. Consequently future events have very little ability to influence what people do now when there is a significant rate of return on capital.
A modern illustration of this effect is found near here at Stanford University. When Leland Stanford left the land on which Stanford University stands to the university, he included a stipulation that the land could never be sold. In response the university has simply leased the lands to Xerox and other corporations for 999 years. This would reduce the sum received, compared to an outright sale, by 0.000000000015% (assuming a modern real interest rate of a meager 3%). Leland Stanford’s stipulations simply have no binding effect on the actions of Stanford University now.

Similarly at even a moderate interest rate such as 3% it is possible for anyone who wishes to do so to found a major university in their own name, as long as they are prepared to wait long enough. Thus if you were to leave $1000 in your will, and specify that it was to be kept in the bank earning interest for 500 years, and then taken out and used to endow a university in your name you would have accumulated $2.6 billion, enough to endow a modest sized university. If you were willing to wait 600 years you would have $50 billion, more than any university currently has in its endowment. One person who early on realized the power of compound interest was Benjamin Franklin who in his will when he died in 1790 left $1000 for the poor of Philadelphia and Boston, but with the stipulation that it was to be kept in the bank for 200 years before it was distributed. When the money was finally distributed in 1990 charities for the poor in each of these cities received $5 million.

The level of the rate of interest can thus effect even simple thinks like how much of the land in a society is left as woodland, and how much is cleared for farmland. Suppose for example that an acre of farmland (producing grain) rents for \( v \) per year. Suppose also that an acre of trees cut for timber yields an amount \( p \), but that it takes 25 years for the trees to grow to maturity. The cost of producing a crop of timber in present value terms is thus,
since the land cannot be used to grow grain for each of these 25 years. The future rents all have
to be discounted to equal dollars of this year. The value of the timber produced will be

\[
\frac{p}{(1+r)^{25}}
\]

Since the timber is not sold till 25 years from now, and the money received has to be discounted
to the present. To make it profitable to plant timber the value of timber relative to grain has to
rise until

\[
\frac{p}{(1+r)^{25}} = \frac{v}{r} \left[ 1 - \frac{1}{(1+r)^{25}} \right]
\]

\[
\frac{p}{v} = \frac{1}{r} \left[(1+r)^{25} - 1 \right]
\]

As \( r \) gets larger, \( p/v \) has to be correspondingly larger before anyone finds it profitable to
plant trees. The way this happens is through the supply of timber falling so that wood becomes
scarcer relative to grain. Suppose that the demand curve for wood is such that
where \( Q \) is the quantity of wood produced per year. In this case the relationship between the interest rate and the quantity of woodland grown in the economy will be as shown in figure 1. As can be seen as the interest rate moves from 0\% to 15\% with this specification of the demand function, the amount of wooded area drops by 88\%. Thus if interest rates were higher in somewhere like England before 1350 it should show up in such ways as, for example, deforestation.\(^1\)

\[
p = \frac{1}{Q}
\]
FIGURE 1: WOODLAND AREA AS A FUNCTION OF THE INTEREST RATE
What Determines the Rate of Return?

The choice between consumption now and consumption next year can be presented in economics by the indifference curve diagram in figure 2. Suppose the consumer has a budget to allocate between consumption in this year or the next. This is shown as the line

\[ C_0 + C_1 = B \]

If there was no time preference, that is if consumption now was equally valued with future consumption, and if the prices of goods were the same this year as the next, the highest indifference curve would be attained on the 45° line where \( C_0 = C_1 \). But in practice people given such a budget constraint choose more consumption now and less later (\( C_1 < C_0 \)). That is they have a preference for consumption in the present, that we call time preference. They only equalize consumption between now and the future if the budget constraint is instead

\[ (1+r)C_0 + C_1 = B_1 \]

That is the price of current consumption is higher by a factor, \( r \), the interest rate or “rate of return”, so that they can consume more if they wait a year. \((1+r)\) is also called the discount rate because it is the amount by which future consumption is “discounted” relative to present consumption.

But while economists have noted this phenomena they have never explained why people display time preference in the way that they do. The rate of return for safe investments has been about 3% since the 1720s in England. But why this number is 3% as opposed to 0% or 1% has never been explained.
Figure 2: Time Preference and Consumption

\[ c_0 + c_1 = B \]

\[ c_1 = c_0 \]

\[ (1+r)c_0 + c_1 = B_1 \]
Risks and Returns

Why were rates of return so high in early societies? This is a very puzzling question. First we need to consider whether there were objective features of the environment of the medieval economy which made interest rates high. That is, were medieval people just people like you or me who faced economic constraints that forced interest rates up? It is a truism of modern investment theory that there is a tradeoff between risks and returns. Stocks yield a higher average return than bonds, but bonds have a lower variance in earnings. Short term bonds yield less than long term bonds, but again short term bonds have less variance in earnings. Were medieval returns so high just because all investments were so risky?

There are two types of risk. The first risk is that the borrower would default or the obligation would be lost. This risk depends on a number of factors including the general level of the stability of an economy. In some societies all investments are very insecure because they are dependent on an unstable political regime. If the government changes the investment can become worthless. Thus companies investing in Russia at present demand a very high rate of return because of three sorts of risks. First the government may change and the new regime may freeze the assets of foreign countries or refuse to honor contracts. Secondly the Russian legal and political system is so corrupt that companies may be unable to enforce their contracts even without any political change. Finally the direction the Russian economy will take is very unclear. It might grow strongly, it might decline.

If the force impeding investment and innovation is the fear of confiscation of property, then people will only risk resources in investment where the rate of return is high enough to compensate them for these risks. Thus the Mexican Revolution of 1910-17 created a long period
of uncertainty in the Mexican economy. Though industrial capital was largely undamaged by the fighting, there was great uncertainty as to the property rights which would prevail in the end. Investment largely ceased and share values plummeted.\(^2\) At the end of the English Civil war in 1650 the victorious parliament sold most of the deposed King’s estates. The perpetuities owned by the crown sold in 1650 for an average implied rate of return of 11.2%, at a time when private perpetuities yielded a return of about 5.5%.\(^3\) The huge premium in returns available to investors in the seized Royal property reflected the political uncertainty that attached to these property rights.\(^4\) The political uncertainties of the hyperinflation period in Weimar Germany led to a dramatic decline in the value of equity, even though this was a real asset, and the economy was experiencing full employment of resources. The real value of shares dropped to 2.7% of their 1913 level by October 1922, and were so low that the auto maker Daimler with its 3 large works as well as large land holdings and technical know-how was valued at 327 of its own cars.\(^5\)

It is hard to measure directly the risks to investment in pre-industrial society from political instability and warfare. From a distance the series of wars and insurrections makes the period seem turbulent. But, for reasons that will be discussed in class, it seems that political risks cannot explain high interest rates in early societies.

The second type of risk is that the lender will not survive to enjoy the fruits of their abstinence. The higher the risk of death of the lender in any year the higher will be the rate of return required. A high risk of death will cause people to "eat, drink, and be merry" now.\(^6\)

\(^2\) See Haber(1989), pp. 122-149.
\(^3\) On crown perpetuities (fee-farm rents) see Madge (1938), p. 237. See figure 5 below for private perpetuities.
\(^4\) These properties were recovered by the crown in 1660. The Parliament was prepared to sell these perpetuities even though they commanded a low price as a means of giving a wide class of people an interest in the survival of the new regime.
\(^5\) Bresciani-Turroni (1937), pp.
\(^6\) This effect will only work in the absence of a financial instrument called an "annuity." An Annuity is a fixed income paid to someone every year in return for a capital sum paid now, which payment ceases at the death of the
Because of the lower life expectancy the chances of dying in any year for someone in the middle ages would be greater than for a modern person. They would not, however, be much higher than the risks of death in any year for people in the seventeenth century, or the eighteenth century. Life expectancy at birth in 1650 was only about 35. By 1800 it had crept up to about 37. This is much lower than modern life expectancies yet in England by 1650 the real rate of return was as low as 5%, and by 1750 it was 3.5%. Thus at maximum 0.5% - 2% of the higher interest rate of the pre-industrial period could be accounted for by the risks of death.

**Income Growth**

Three other factors might explain the high medieval interest rates. First there is the rate of economic growth measured as increases in income per person. Also there is the level of income in the society, and the income distribution.

First consider why there is a positive real interest rate of about 3% in most modern developed economies? The existence of a positive interest rate implies that if we are prepared to delay consuming then we could all consume much more. Why are people not all persuaded to wait to consume at a 3% interest rate so that they could consume much more? If everyone did this it would increase the supply of capital and drive down the rate of return. Several things cause people to postpone consumption even given a positive real interest rate. The first is the expectation that income will grow. If I expect a higher income in future then $1 now can be worth more to me than $2.4 in 30 years. One type of society in which we would expect a high purchaser. The higher the risk of death the higher will be the rate of return paid on the annuity, since the expected payment in each future year will be lower. The higher return on annuities when life is riskier will compensate the lender for the risk, and induce the same amount of lending as would occur with no death risk.
rate of return thus would be a society which was experiencing rapid economic growth so that real incomes were increasing year by year.

Interest rates in the Central Valley in California in the 1850s, for example, were in the order of 50% per year, even though interest rates at that time in England were about 2.5%. At this stage California was remote from other capital markets and was experiencing rapid economic growth. Many settlers were arriving and investing large amounts of resources in forming farms. Their current consumption was low because of the large investment effort they were making. But they expected much higher incomes in a few years. Thus even a 50% interest rate could not persuade people to save more and consume less. Even though at this interest rate every $1 they saved would be worth $2 in less than two years.

Conversely, we would then expect that in a static society with no income growth the rate of interest would be lower because this force would not be operating to push up interest rates. Interest rates could not be high because people would expect their future income to be the same as their current income. This would argue for low interest rates in the medieval period.

Poverty

The third objective circumstance we have to consider is that people were poor in medieval society. Would this depress the interest rate, by making people less prepared to wait for future consumption because the needs of the present are so great? There is plenty of evidence in modern America that poorer people have what is called a "higher rate of time preference." What is meant by this is that they are willing to pay higher interest rates to borrow than richer people, and need to be paid more to defer consumption than richer people. A survey of people's rate of time preference conducted in 1972 found the following result:
The rate of time preference was determined by asking people questions such as "If you could have $100 today or $120 in one year's time which would you prefer?" Clearly poorer and less educated people reveal higher rates of time preference, though the rates revealed by everyone are high. There is a problem of causation in interpreting these results though. Those with the highest rates of time preference are the least likely to get education and to invest in job training that will later yield them higher incomes. Thus the connection of time preference rates with income and education in cross section in society may not come because income levels determine the rate of time preference.7

There is no reason in economic theory to expect that people at lower incomes would have higher time preference rates. If you are poor, and know that you are going to remain so, then even though the needs of the present are pressing so are the needs of the future, so that your relative valuation of consumption now and in the future should not change.

If income was a major determinant of time preference then if we graph real wages in England against the rate of return from 1200 to 1850 by half centuries we should see a

<table>
<thead>
<tr>
<th>Earnings of “head” of family</th>
<th>8 years or less of education</th>
<th>16 years or more of education</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2,000</td>
<td>58%</td>
<td>41%</td>
</tr>
<tr>
<td>$15,000</td>
<td>35%</td>
<td>18%</td>
</tr>
</tbody>
</table>

7There is also a problem that some of those with low incomes will have those incomes only temporarily (for example, because they are students, or have lost a job), and consequently expect higher incomes later. In this case they will have a higher rate of time preference because they will want to borrow against some of that future income to smooth out their consumption over time.
correlation between the two. Figure 3 shows the one graphed against the other. As can be seen
the figure is largely ambiguous on this issue. If we exclude the earliest years from 1200-1349
there is no sign of any connection between interest rates and wage levels. But it turns out real
incomes were at their lowest in the years before the plague. And here we observe very high real
interest rates. So one possible explanation of the very high interest rates in early Europe which
we cannot rule out is the very low level of real incomes before 1349.

FIGURE 3: THE RATE OF RETURN IN ENGLAND VERSUS THE REAL WAGE
INEQUALITY AND THE RATE OF RETURN ON CAPITAL

The last possible "objective" source of differences in the rate of return over time would be the degree of inequality in income. It is well known that the savings rate of people varies by level of income. Poor people typically have very low savings rates. They consume almost all their current income. In the nineteenth century laborers in England typically had just such a 0 savings rate, so that when they got too old to work they had to be supported by their children or by "Poor Relief" from the local parish. Richer people have much higher savings rates. Cipolla gives many examples of their higher savings rates in the pre-industrial period. The richest families in Italy in the late 16th and early 17th century were saving 70-80% of their income annually. One of the richest merchants in Amsterdam in the late 17th century similarly saved 50-70% of his income. In this case a crucial determinant of the supply of capital will be the distribution of income in the society. The more equal the income distribution at a given level of average income per capita, the less will be the supply of savings, and the higher will be the rate of return. What happened to income inequality over time in a country such as England? This question is hard to answer because in part of the paucity of data, but also because of the important role of the Church up till 1535 and the Dissolution of the Monasteries by Henry VIII, where much of the land of the church was claimed by the crown. How should we classify the income of the Church in the distribution of income?

Table 2 shows the information we have on the distribution of income in England circa 1300. As can be seen the King received a very high income relative to an unskilled worker. But the King spent much of his income for government functions such as the interminable small scale wars of the medieval period. The King's income at about 24,000 times the unskilled wage would
correspond in the USA now to an income of about $240 m per year, and to a fortune of about $3 b (about the wealth of Ross Perot). Since Bill Gates has a fortune of 50-80 billion the English King was thus relatively poorer than some of the richest people now. The top 1% of the secular population seemed to earn about 13% of all secular income (the income of the Church which was enjoyed by a small group of priests and nuns would have to be added to this to get an idea of the overall share enjoyed by the top 1 or 2%).

**TABLE 2: INDICATIONS OF INCOME INEQUALITY IN ENGLAND, 1300**

<table>
<thead>
<tr>
<th>Group</th>
<th>N</th>
<th>Income</th>
<th>Multiple of Laborer’s Income</th>
</tr>
</thead>
<tbody>
<tr>
<td>King</td>
<td>1</td>
<td>£30,000</td>
<td>24,000</td>
</tr>
<tr>
<td>Earls</td>
<td>18</td>
<td>£400-11,000</td>
<td>320-9,000</td>
</tr>
<tr>
<td>Barons</td>
<td>136</td>
<td>£200-500</td>
<td>160-400</td>
</tr>
<tr>
<td>Knights</td>
<td>1,100</td>
<td>£40-200</td>
<td>32-160</td>
</tr>
<tr>
<td>'Lesser Gentry'</td>
<td>10,000</td>
<td>£5-40</td>
<td>4-32</td>
</tr>
<tr>
<td>Peasants,</td>
<td>750,000</td>
<td>£1.25-5</td>
<td>1-4</td>
</tr>
<tr>
<td>Craftsmen</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Laborers</td>
<td>350,000</td>
<td>£1.25</td>
<td>1</td>
</tr>
</tbody>
</table>

The next benchmark for which we get an estimate of the degree of inequality in income is 1436. In 1436 an income tax was levied which gives some idea of the distribution of income again. Table 3 shows the distribution of incomes at this time. Now the top 1.4% of the secular population enjoys about 14% of secular income, which implies a slight move towards equality since 1300. We have information on church incomes by this period (extrapolating back from the valuations made at the Dissolution of the Monasteries in 1535) which suggests that the Church income would exceed that of the rich secular society. Thus counting both the nobles and the Church, about 2% of the population in 1436 enjoyed about 25% of income. But there is no reason to believe that this percentage was any less in 1300. Thus the dramatic decline in rates of return from 1350 to 1450 seemingly took place without any fundamental shift in the distribution of income.
### TABLE 3: INDICATIONS OF INCOME INEQUALITY IN ENGLAND, 1436

<table>
<thead>
<tr>
<th>Group</th>
<th>N</th>
<th>Income</th>
<th>Multiple of Laborer's Income</th>
</tr>
</thead>
<tbody>
<tr>
<td>Secular</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>King</td>
<td>1</td>
<td>?</td>
<td></td>
</tr>
<tr>
<td>Earls</td>
<td>18</td>
<td>&lt; £4,000</td>
<td>&lt; 1,600</td>
</tr>
<tr>
<td>Barons</td>
<td>70</td>
<td>£300-2,500</td>
<td>120-1000</td>
</tr>
<tr>
<td>Knights</td>
<td>933</td>
<td>£20-400</td>
<td>8-160</td>
</tr>
<tr>
<td>'Gentlemen'</td>
<td>6,000</td>
<td>£10-40</td>
<td>4-16</td>
</tr>
<tr>
<td>Peasants,</td>
<td>340,000</td>
<td>£2.5-10</td>
<td>1-4</td>
</tr>
<tr>
<td>Craftsmen</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Laborer</td>
<td>160,000</td>
<td>£2.5</td>
<td>1</td>
</tr>
<tr>
<td>Clergy</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bishops</td>
<td>17</td>
<td>£400-3,500</td>
<td>160-1400</td>
</tr>
<tr>
<td>Rich Priests</td>
<td>850</td>
<td>&gt; £40</td>
<td>&gt; 16</td>
</tr>
<tr>
<td>Rich Monasteries</td>
<td>126</td>
<td>£300-2,500</td>
<td>120-1000</td>
</tr>
<tr>
<td>Poor Monasteries</td>
<td>700</td>
<td>&lt; £300</td>
<td>&lt; 120</td>
</tr>
<tr>
<td>Poor Priests</td>
<td>7,650</td>
<td>£5-40</td>
<td>2-16</td>
</tr>
</tbody>
</table>

**Source:** Dyer (1989), pp. 30-33.
We get another benchmark on income inequality from the tables constructed by the Political Arithmetician Gregory King for England in 1688. According to King's estimates the top 2% of families would earn about 21% of income in 1688. Thus there was a narrowing of income inequality at the top from 1436 to 1688. We can compare the degree of income inequality in these distributions by considering what share of income the top few percent of the population earned compared to the modern economy. These figures suggest the following:

<table>
<thead>
<tr>
<th>Period</th>
<th>Share Earned by Top 2%</th>
<th>Share Earned by Top 1%</th>
<th>Share Earned by Top 1% (Secular)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1300</td>
<td>-</td>
<td>-</td>
<td>13%</td>
</tr>
<tr>
<td>1436</td>
<td>25%</td>
<td>-</td>
<td>12%</td>
</tr>
<tr>
<td>1688</td>
<td>21%</td>
<td>13%</td>
<td>-</td>
</tr>
</tbody>
</table>

These figures suggest that the degree of income equality was fairly stable in England from 1300 to 1688. The big decline in rates of return between these periods cannot then have been caused by any change in the income distribution.
Chapter Problems

1. Suppose for medieval France you have the following data:

<table>
<thead>
<tr>
<th>Asset</th>
<th>Units</th>
<th>Price</th>
<th>Annual Income</th>
</tr>
</thead>
<tbody>
<tr>
<td>Land</td>
<td>Acre</td>
<td>40 livres</td>
<td>4 livres</td>
</tr>
<tr>
<td>Houses</td>
<td>-</td>
<td>100 livres</td>
<td>13 livres</td>
</tr>
<tr>
<td>Rent Charges</td>
<td>-</td>
<td>50 livres</td>
<td>6 livres</td>
</tr>
<tr>
<td>Life Annuities</td>
<td>1 life</td>
<td>50 livres</td>
<td>10 livres</td>
</tr>
<tr>
<td>Crown Loans</td>
<td></td>
<td>100 livres</td>
<td>25 livres</td>
</tr>
</tbody>
</table>

Suppose also the inflation rate in general is 1%, and the prices of land and houses are increasing at 3% per year.

(a) Calculate the gross nominal rate of return on each asset.
(b) Calculate the gross real rate of return on each asset.
(c) What is the best indicator of the risk free long term real interest rate?

2. Suppose the interest rate is 10% and a piece of land rents for £10 a year.

   (1) How much would you pay for the land outright assuming no inflation in general prices or land prices?
   (2) How much would the land sell for if the owner gets it back in 100 years?
   (3) How much would the land sell for if the owner gets it back in 21 years?
3. Suppose land rents for £10 a year. Suppose also a full grown crop of timber on the land will fetch £300, but takes 25 years to mature. Will land be used for timber at

(a) A 10% interest rate?
(b) A 5% interest rate?
(c) A 0% interest rate?
(d) Calculate the interest rate at which the land will be equally profitably employed for timber or for farmland.
(e) Suppose the interest rate is 10%. At what price for timber will any timber start to be grown.
(f) Explain what will happen in the marketplace to ensure that timber sells for this price.

4. In 1650 the estates of the King, who had been executed in 1649 were sold off to raise revenue. Included in the sales were rights to receive rent charges in perpetuity. At this time in England private rent charges sold for a 5.5% rate of return. Thus a £5 rent charge would sell for £90.91. But a £5 rent charge belonging to the crown typically sold for only £44.64.

(a) What was the rate of return on the crown rent charges?
(b) Why was it different from the rate of return on private rent charges?
(c) The crown rent charges were confiscated from the purchasers in 1660 when the son of the King was restored to the throne. Did the people who bought crown rent charges in 1660 make a profit or loss from their purchase, and what was the amount of the profit or loss in 1650 money?
1.)

(a) **Calculate the gross nominal return on each asset.**

The gross nominal return of an asset is defined as the annual payment received as a percentage of the initial investment. \( r_N = \frac{R}{V} \) where \( r_N \) is the nominal rate of return, \( R \) is the annual payment, and \( V \) is the price of the asset.

Thus, the nominal rates of return on each asset are as follows:

- Land: \( r_N = \frac{4}{40} = .10 \)
- Houses: \( r_N = \frac{13}{100} = .13 \)
- Rent Charges: \( r_N = \frac{6}{50} = .12 \)
- Life Annuities: \( r_N = \frac{10}{50} = .20 \)
- Crown Loans: \( r_N = \frac{25}{100} = .25 \)

(b) **Calculate the gross real rate of return on each asset.**

The real rate of return on an asset is defined as the nominal rate of return minus the rate of inflation. Therefore, the real rate of return gives a measure of the change in purchasing power due to holding a given asset to maturity. When the price of the asset in question rises at the same rate of inflation as the general price level, then the equation for the real rate of return is \( r_r = \frac{R}{V} - \pi \). Where, \( r_r \) is the real rate of return, \( R \) and \( V \) are as above, and \( \pi \) is the general inflation rate.

When the price of the asset in question changes at a different rate than the general price level, however, the equation must be altered to reflect this fact. If the price of an asset rises faster than the general price level, then when the asset is sold, the purchasing power of the asset holder will have increased by more than would be indicated by the above equation. An increase in the price of an asset will add to the purchasing power of the asset holder, so the difference between the inflation rate of the specific asset and the general inflation rate must be added to the nominal rate of return to arrive at the correct measure of the change in purchasing power.

Thus \( r_{ri} = \frac{R}{V} + (\pi_i - \pi) \), where \( r_{ri} \) is the real rate of return on asset \( i \), and \( \pi_i \) is the rate of inflation for asset \( i \).
Land: \( r_r = \frac{4}{40} + (.03 -.01) = .12 \)

Houses: \( r_r = \frac{13}{100} +(.03 - .01) = .15 \)

Rent Charges: \( r_r = \frac{6}{50} - .01 = .11 \)

Life Annuities: \( r_r = \frac{10}{50} - .01 = .19 \)

Crown Loans: \( r_r = \frac{25}{100} -.01 = .24 \)

(c) What is the best indicator of the risk free long-term real interest rate?

In the long run, it is expected that the rate of inflation on land will equal the general rate of inflation in the economy, therefore, the real rate of return on land will equal the nominal return. Also, of the assets listed, land is the least risky. Therefore, the nominal rate of return on land is the best indicator of the long-term risk free rate of return.

2.) Suppose the interest rate is 10% and a piece of land rents for £10 a year.

(a) How much would you pay for the land outright assuming no inflation in general prices or land prices?

The price you would be willing to pay for a piece of land will be equal to the present value of the infinite stream of rental income to which ownership entitles you:

\[
P = \frac{v}{1+r} + \frac{v}{(1+r)^2} + ... + \frac{v}{(1+r)^n} + ... = \sum_{i=1}^{\infty} \frac{v}{(1+r)^i} = \frac{v}{r}
\]

Where \( v \) = annual rental payment, and \( r \) is the interest rate.
So \( P = £10/.10 = £100 \).

(b) How much would the land sell for if the owner gets it back in 100 years?
The price of a 100 year lease will be equal to the present value of the sum of 100 annual rental payments:

\[
P_{100} = \frac{v}{(1+r)} + \frac{v}{(1+r)^2} + \ldots + \frac{v}{(1+r)^{100}}
\]

\[
P_{100} = \sum_{i=1}^{\infty} \frac{v}{(1+r)^i} - \sum_{i=100}^{\infty} \frac{v}{(1+r)^i} = \frac{v}{r} \left( 1 - \frac{1}{(1+r)^{100}} \right)
\]

Therefore, in this case:

\[
P_{100} = \frac{10}{.10} \left( 1 - \frac{1}{(1.10)^{100}} \right) = 99.99
\]

(c) How much would the land sell for if the owner gets it back in 21 years?

The price of a 21 year lease is calculated as above:

\[
P_{21} = \frac{v}{r} \left[ 1 - \frac{1}{(1+r)^{21}} \right] = \frac{10}{.10} \left[ 1 - \frac{1}{(1.10)^{21}} \right] = 86.48
\]

3.) Suppose land rents for £10 a year. Suppose also a full grown crop of timber on the land will fetch £300, but it takes 25 yeas to mature. Will land be used for timber at

(a) A 10% interest rate?

In order to answer this, we must calculate the discounted benefits of each activity.

The benefits of growing timber\(B_T\) will be a £300 lump sum payment in 25 years which will have a present value of \(B_T = \frac{300}{(1+r)^{25}}\).

The benefits of renting the land \((B_R)\) be the sum of the annual interest payments over the next 25 years, discounted at interest rate \(r\). This will be equivalent to the value of a 25 year lease, calculated as above: \(B_R = \frac{v}{r} \left[ 1 - \frac{1}{(1+r)^{25}} \right]\).

When \(r = .10\),
Therefore, the discounted benefits of renting the land are greater than the discounted benefits of growing timber, and the landowner will opt to rent the land.

(b) **What if r = 5%?**

Calculate and compare the discounted benefits at an interest rate of 5%.

\[
B_T = \frac{300}{(1.05)^{25}} = 88.59 \quad \text{and} \quad B_R = \frac{10}{0.05} \left[ 1 - \frac{1}{(1.05)^{25}} \right] = 140.94
\]

So, once again, the landowner will rent the land.

(c) **A 0% interest rate?**

Since a zero percent interest rate implies future payments are not discounted at all, then the benefits to using the land for timber will simply be the £300 future payment, and the benefits to renting will be the sum of the 25 annual £10 rental payments.

\[
B_T = 300 \quad \text{and} \quad B_R = (25)(£10) = £250.
\]

Thus, a zero percent interest rate will induce landowners to use their land for timber.

(d) **Calculate the interest rate at which the land will be equally profitably employed for timber or for farmland.**

We want to solve for the interest rate which will satisfy the following equality:

\[
\frac{300}{(1 + r)^{25}} = \frac{10}{r} \left[ 1 - \frac{1}{(1 + r)^{25}} \right]
\]

\[
\frac{300}{10} = (1 + r)^{25} \left[ 1 - \frac{1}{(1 + r)^{25}} \right]
\]

\[
30r = (1 + r)^{25} - 1
\]

\[
(1 + r)^{25} - 30r - 1 = 0
\]

The easiest way to solve this is to use a grid search on a spreadsheet. We know from above that the critical value for \( r \) must be between 0% and 5%, so start by plugging in all values of \( r \) in this interval into the above equation, and find a new interval that is bounded by the last value for \( r \) such that the above expression is positive, and the first value for which it is negative. Add one more decimal place to your values for \( r \), and repeat until you are satisfied with the level of precision. The value I found was approximately 1.484%. Please note that you will not be expected to solve a problem like
this on an exam, but it is important to recognize the existence of such a critical value for \( r \), and how to set up the above problem.

(e) Suppose the interest rate is 10%. At what price for timber will any timber start to be grown?

Here we want to find the value of the price of timber such that a landowner will be indifferent between growing timber and renting the land, when \( r = 10\% \). Therefore, we must solve the following equality for \( P_T \) (the price of timber).

\[
\frac{P_T}{(1.10)^{25}} = \frac{10}{.10} \left[ 1 - \frac{1}{(1.10)^{25}} \right]
\]

\[
P_T = 100(1.10)^{25} \left[ 1 - \frac{1}{(1.10)^{25}} \right] = 983.47
\]

(f) Explain what will happen in the marketplace to ensure that timber sells for this price.

As seen from problems (a) through (c) above, there is a negative relationship between the interest rate and the quantity of timber grown, regardless of the price. Therefore, at a high interest rate, timber production will be insufficient to meet demand, thus driving the price up until a new equilibrium is reached.

4.)

(a) What was the rate of return on the crown rent charges?

The rate of return is equal to the annual payment divided by the price of the asset:

\[
r = \frac{5}{44.64} = 11.2\%.
\]

(b) Why was it different from the rate of return on private rent charges?

The rate of return on crown rent charges is much higher to reflect the fact that buyers must be compensated for the risk of having their assets seized in the event of a restoration of royal power. In other words, no rational consumer would buy a crown rent charge if it had a similar rate of return as that of the much less risky private rent charge.

(c) The crown rent charges were confiscated from purchasers in 1660 when the son of the King was restored to the throne. Did the people who bought crown rent charges in 1650 make a profit or loss from their purchase, and what was the amount of the profit or loss in 1650 money?
In order to make this determination, we must calculate the discounted value of the annual payments actually received between 1650 and 1660, and compare this to the price paid in 1650 of £44.64.

We can calculate this sum as if it were the price of a ten year lease, discounted at the interest rate of 5.5%. We use 5.5% as the discount factor because it is the rate of interest paid on the alternative asset available in the economy.

\[
\text{Discounted revenue} = \frac{5}{0.055} \left[1 - \frac{1}{(1.055)^{10}}\right] = 37.69 < 44.64
\]

We may conclude therefore, that the investors made a loss of £44.64 - £37.69 = £6.95.