PROBLEM SET #5: INDIVIDUAL CHOICE AND DEMAND CURVES - ANSWERS

Individual Demand - The Indifference Curve Interpretation

1. Slobodon spends all his income on meat or on potatoes. Draw his budget constraint in each of the following cases.

   a. His income is $100. Meat costs $2 per pound, potatoes cost $1 per pound.
   b. His income is $100. Meat costs $4 per pound, potatoes cost $1 per pound.
   c. His income is $200. Meat costs $4 per pound, potatoes cost $1 per pound.
   d. His income is $200. Meat costs $8 per pound, potatoes cost $2 per pound.
2. Use indifference curves to illustrate your preferences between the following goods.

   a. Left and right shoes. What do we call such goods?

   These are called “Perfect Complements” because there exists a fixed proportion that they are always consumed in (here 1 left shoe for every 1 right shoe). The indifference curves are below.
b. Quarters and nickels. What do we call such goods?

“Perfect Substitutes.” This means that you always trade them in a fixed proportion. In this case you always exchange five nickels for one quarter. The indifference curves are below.
c. Between chocolate and copies of Das Kapital (in German).

These goods will generate “lexicographic preferences.” The term lexicographic should remind you of putting things in abc order. The idea here is the average person gets no utility from a copy of Das Kapital (which is, by the way, Karl Marx’s famous treatise on the role of economic class on society’s development, so you should get some utility from it). So once you have 1 piece of chocolate, you don’t care whether you have 1, 2, or a million copies of the book. And you’d rather have 2 chocolates and no books than 1 chocolate and a million copies of the book. The indifference curves look like this:

![Diagram showing indifference curves between chocolate and copies of Das Kapital]
3. Suppose that the Borgia family has $100 to spend on food per day. Suppose also that they always use 1 lb. of salt per 999 lbs of food they consume.

   a. What kind of goods are salt and food for the Borgias.

   Clearly salt and food are “perfect complements” since they are always consumed in a fixed proportion to one another.

   b. If the price of food is $p_f$ and the price of salt $p_s$ show the budget constraint for the Borgia’s.
c. Suppose that initially a lb. of salt costs $1, and a lb. of food $1. How many pounds of salt and how many pounds of food are consumed?

Given \( p_s = 1, p_f = 1 \). You need to solve the following 2 equations simultaneously

1. \( 1.100 = (1 \cdot Q_f) + (1 \cdot Q_s) \)
2. \( Q_f = 999 \cdot Q_s \)

The first equation is the budget constraint. The second equation imposes the restriction of the fixed proportion you are told that food and salt are always consumed in. Solving these yields:

\[
Q_s = 0.1 \\
Q_f = 99.9
\]

d. Suppose the price of salt falls to $0.50 per pound. What is the new budget constraint? How much food and how much salt are consumed?

Given \( p_s = .5, p_f = 1 \). You need to solve the following 2 equations simultaneously

1. \( 100 = (1 \cdot Q_f) + (.5 \cdot Q_s) \)
2. \( Q_f = 999 \cdot Q_s \)

Solving these yields:

\[
Q_s = 0.10005 \\
Q_f = 99.95
\]

e. What is the price elasticity of demand for salt?

Calculate the price elasticity of demand for salt using the following previously calculated information

- when \( p_s = 1, Q_s = .1 \)
- when \( p_s = .5, Q_s = .10005 \)

Elasticity then equals:

\[
\frac{(0.10005 - 0.1)}{(0.5 - 1)} \cdot \frac{1}{0.1} = -0.001
\]

Of course we ignore the negative sign and report a price elasticity of 0.001
f. Suppose instead the price of food falls to $0.50 per lb. and the price of salt is still $1. What is the consumption of food and salt now?

Given ps = 1, pf = .5

You need to solve the following 2 equations simultaneously

1. 100 = (.5*Qf) + (1*Qs)
2. Qf = 999*Qs

Solving these yields

Qs = 0.1998
Qf = 199.6


g. What is the price elasticity of demand for food? Why is the answer here so different?

Now the price elasticity of the demand for food is

\[
\frac{(199.6 - 99.9)}{(.5 - 1)} \times \frac{1}{99.9} = -2
\]

Again we ignore the negative sign and report an elasticity of 2.
Demand Elasticities

4. Suppose demand for a good is given by \( Q = 100 - \frac{P}{2} \).

   (a) Draw the demand curve.

\[ Q = 100 - \left(\frac{P}{2}\right) \]

(b) Calculate \( E_d \) when \( p = 0, p = 100, \) and \( p = 200 \) (using the formula for demand at a given point on the curve).

Note that the slope of the demand curve is 2 (not .5!)

\[ E(\frac{1}{\text{slope}}) \times \left(\frac{P}{Q}\right) \]

\[ P = 0 \quad P = 100 \quad P = 200 \]

\[ E = 0.5 \times \left(\frac{0}{100}\right) \quad E = 0.5 \times \left(\frac{100}{50}\right) \quad E = 0.5 \times \left(\frac{200}{0}\right) \]

(c) What price maximizes the revenue received by the seller? (Quick way – use calculus. Slow way – find the price by trial and error).

It can be shown by differentiating the revenue function \( P \times Q \) with respect to \( P \) that \( P = 100 \) maximizes the revenue in this case.

By trial and error:
P=50 implies Q=75 implies Revenue = 3,750  
P=100 implies Q=50 implies Revenue=5,000  
P=150 implies Q=25 implies Revenue=3,750

And any other values of P you try will lead to revenues less than 5,000. So P=100 maximizes revenue.

(d) What is $E_d$ at this revenue maximizing price? Is this just a coincidence?

So we know that the elasticity of demand equals one at this price. Clearly this isn’t a coincidence revenue is maximized at the price where price elasticity 1.
Income Elasticities

5. The table below shows family income in $ versus average food consumption (in $), housing consumption per person (in ft²), automobile consumption (in $), and public transportation consumption (in $).

<table>
<thead>
<tr>
<th>Income ($)</th>
<th>Food consumption ($)</th>
<th>Housing (ft²)</th>
<th>Autos ($)</th>
<th>Public Transport ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10,000</td>
<td>5,000</td>
<td>300</td>
<td>100</td>
<td>500</td>
</tr>
<tr>
<td>10,100</td>
<td>5,020</td>
<td>303</td>
<td>104</td>
<td>498</td>
</tr>
<tr>
<td>30,000</td>
<td>9,000</td>
<td>900</td>
<td>1,400</td>
<td>340</td>
</tr>
<tr>
<td>30,100</td>
<td>9,010</td>
<td>903</td>
<td>1,412</td>
<td>339</td>
</tr>
</tbody>
</table>

(a) Calculate the income elasticity of demand (E_i) for each of the goods as income goes from $10,000 to 10,100 and as income goes from $30,000 to $30,100.

Note here that we arbitrarily choose to base our elasticity calculations around the INITIAL level of income, rather than the NEW level. You will get slightly different answers depending on which point you use as a reference point. But there is no way to say which is more precise, they are both approximations.

From 10,000 to 10,100

<table>
<thead>
<tr>
<th></th>
<th>Food</th>
<th>Housing</th>
<th>Autos</th>
<th>Public Transport</th>
</tr>
</thead>
<tbody>
<tr>
<td>Change in quantity</td>
<td>20</td>
<td>3</td>
<td>4</td>
<td>-2</td>
</tr>
<tr>
<td>Change in income</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>Income</td>
<td>10,000</td>
<td>10,000</td>
<td>10,000</td>
<td>10,000</td>
</tr>
<tr>
<td>Quantity</td>
<td>5,000</td>
<td>300</td>
<td>100</td>
<td>500</td>
</tr>
<tr>
<td>Elasticity</td>
<td>0.4</td>
<td>1.00</td>
<td>4.00</td>
<td>-0.4</td>
</tr>
</tbody>
</table>

From 30,000 to 30,100

<table>
<thead>
<tr>
<th></th>
<th>Food</th>
<th>Housing</th>
<th>Autos</th>
<th>Public Transport</th>
</tr>
</thead>
<tbody>
<tr>
<td>Change in quantity</td>
<td>10</td>
<td>3</td>
<td>12</td>
<td>-1</td>
</tr>
<tr>
<td>Change in income</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>Income</td>
<td>30,000</td>
<td>30,000</td>
<td>30,000</td>
<td>30,000</td>
</tr>
<tr>
<td>Quantity</td>
<td>9,000</td>
<td>900</td>
<td>1,400</td>
<td>340</td>
</tr>
<tr>
<td>Elasticity</td>
<td>0.33</td>
<td>1.00</td>
<td>2.57</td>
<td>-0.88</td>
</tr>
</tbody>
</table>
Note $E(\text{income}) = \frac{\text{change in quantity}}{\text{change in income}} \times \frac{\text{income}}{\text{quantity}}$

(b) Which of the goods are normal and which inferior?

Food, housing and autos are normal goods because they have income elasticities that are positive. Public Transport is an inferior good because its income elasticity is negative.

(c) Which of the goods are luxuries and which necessities?

Autos are a luxury because they have an income elasticity greater than 1. Food and Public transport are necessities because they have income elasticities less than 1. Housing is neither inferior nor necessary because its income elasticity is exactly 1.

(d) What is the connection between the income elasticities and what happens to the share of income spent on each good as incomes rise?

For luxury goods, the share of income spent on the good increases as one’s income increases. For necessary goods, the share of income spent on the good decreases as one’s income increases.

(e) Can you think of a reason why the income elasticity of demand for automobiles would be so different for that of public transport?

We might predict that autos have higher income elasticities than public transport because typically traveling by car saves time. As incomes increase, the opportunity cost of one’s time increases, so there are stronger incentives to spend money on things that save time (like driving instead of taking public transport).

(f) What does the income elasticity of demand for housing imply about the size of future American cities if incomes per capita double in the next 30 years?

If the income elasticity remained at exactly one, we would predict that if people’s incomes double they will exactly double their spending on housing. This may suggest that American cities will grow as people purchase more and/or larger houses. The increased expenditure may, however, take the form of simply living in houses of a higher quality but not necessarily larger.