Directed Search and the Bertrand Paradox

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The Paradox

A situation in which two firms set their price equal to marginal cost

One would expect oligopolistic firms to earn positive profits by charging prices above cost

Critical assumption is that firms compete in prices
Suggested Solutions

Solutions to the Bertrand Paradox (BP) include:

1) Differentiated goods (or heterogenous firms)

2) Dynamic competition

3) Capacity constraints
This Paper

- This paper is closer to the third explanation- capacity constraints: Static game, homogenous good and firms

- Need additional assumptions that clarify what happens in case a firm faces excess demand

- I consider a model which provides micro-foundations on the demand side: Rational buyers observe prices and choose which seller to visit

- The twist is search frictions: costly to visit more than one sellers and buyers cannot coordinate their visiting strategies
The Standard Directed Search Model

- A model of competition in markets that do not perfectly clear
- Sellers have one unit of the good and post prices
- Buyers observe prices and visit one seller
- Buyers have to be smart about which seller to visit
- Buyers do not necessarily visit seller with lowest price
  ⇒ channel through which Paradox fails
My Model

• I augment the baseline model with **three ingredients:**
  
  ▶ general capacity constraint
  ▶ consumption externalities or congestion effects
  ▶ pricing based on ex post realized demand (as in Coles Eeckhout, JET 2003)

• I study the three ingredients in isolation

• I show that under any of the three ingredients the Paradox fails
Contributions

- Extend the Directed Search model to include general capacity
  - Show that the Paradox fails \textit{iff} sellers face \textit{some} capacity constraint
  - Study the relationship between frictions and profitability in the market

- Provide two new stories (ingredients) that lead to failure of the Paradox in a static model, with homogeneous good and sellers

- Compare welfare among various environments and make policy suggestions

- Offer a new perspective of looking at the BP:
  Constrained capacity is just a subcase of setups where the Paradox fails
Related Literature A

- Vast literature in IO describing cases where BP fails

- Examples are:
  - Kreps and Scheinkman (1983) (Capacity Constraints)
  - Shaked and Sutton (1982) (Product Differentiation)
  - Dudey (1992) (Repeated Interaction)
Related Literature B

- Directed Search Literature

- Examples are:
  - Burdett, Shi, Wright (2001)
  - Coles and Eeckhout (2003)
  - Geromichalos (2011)
  - Lester (2009)
  - Lester (2010)
The Model

- $n$ buyers, $m$ sellers, $n, m \geq 2$, all risk neutral

- Buyers are identical; each wishes to purchase one unit

- Good is indivisible and homogeneous

- Sellers are identical; each can produce $i \leq k$ units at $c(i) = ci$

- Buyers can only visit one seller and cannot coordinate visiting strategies

- A buyer who consumes at a seller who serves $i$ customers enjoys $u(i)$
  Assume $h < i$ implies $u(h) \geq u(i)$
The Model (Cont’ed)

- In first stage, sellers post $p^j = (p^j_1, ..., p^j_n)$ taking rivals’ prices as given.

- In second stage, buyers observe all advertisements and choose one seller.

- Will focus on mixed strategies, consistent with inability to coordinate. Equilibria in pure strategies exist, but they have undesirable properties.

- A buyer who visits seller $j$ with $i$ customers has net utility $u(\min\{k, i\}) - p^j_i$.

- Ex post participation requires $p^j_i \leq u(\min\{k, i\})$, for all $i \leq n$. 
Ex Ante Payoffs

- Let seller $j$ announce $p^j$ and get visited by an arbitrary buyer with probability $\theta$

- The expected utility of a buyer who visits seller $j$ is

$$U^j\left(p^j, \theta\right) = \sum_{i=1}^{n} \left(\begin{array}{c} n-1 \\ i-1 \end{array}\right) (1 - \theta)^{n-i} \theta^{i-1} \frac{\min\{i, k\}}{i} \left[u(\min\{i, k\}) - p^j_i\right]$$

- The expected profit of seller $j$ is

$$\pi^j\left(p^j, \theta\right) = \sum_{i=1}^{n} \left(\begin{array}{c} n \\ i \end{array}\right) (1 - \theta)^{n-i} \theta^i \min\{i, k\} \left(p^j_i - c\right)$$
Symmetric Equilibrium in Mixed Strategies

DEFINITION: A subgame perfect equilibrium is a collection of price vectors $p^j$, $j = 1, \ldots, m$ and a strategy $s : \prod_{j=1}^{m} p^j \rightarrow \Delta_m$, such that:

i) Given the posted prices, the strategies $s^i = s$, $i = 1, \ldots, n$ maximize buyers’ expected utility, and

ii) Given buyers’ strategy $s$, $p^j$ is a best response to the prices announced by other sellers, for all $j = 1, \ldots, m$.

$\Delta_m$ denotes the unit simplex

I will focus on symmetric equilibria
A Model with Capacity Constraints

- Assume $k \leq n$, $u(i) = u$, $p^i_j = p^j$

- Under this specification...

- Expected utility of a buyer who visits seller $j$ is

$$U^j(p^j, \theta) = (u - p^j) \sum_{i=1}^{n} \binom{n-1}{i-1} (1 - \theta)^{n-i} \theta^i \frac{\min\{i, k\}}{i}$$

- Expected profit of seller $j$ is

$$\pi^j(p^j, \theta) = (p^j - c) \sum_{i=1}^{n} \binom{n}{i} (1 - \theta)^{n-i} \theta^i \min\{i, k\}$$
The Seller’s Problem

- Seller $j$ solves

$$\max_{p^j} \pi^j \left( p^j, \theta \right)$$

$$s.t. \ U^j \left( p^j, \theta \right) = U^l(\tilde{p}, \tilde{\theta}).$$

- In words, the seller sets $p^j$...

- ...understanding that, in second stage, buyers will observe $p^j, \tilde{p}$...

- ... and will determine probabilities of visiting each seller, such that they are indifferent between seller $j$ or any seller $l \neq j$

- If $\theta$ represents probability of visiting seller $j$, then $\tilde{\theta} = (1 - \theta)/(m - 1)$
**Equilibrium Prices**

**LEMMA 1**

In the unique symmetric equilibrium, every buyer visits each seller with probability $\theta^* = 1/m$, and all sellers announce the price

$$p^*(n, m; k) = \frac{\sum_{i=1}^{n} H(i, n, m) \min\{i, k\}\left\{\frac{m}{m-1} [1 - F(i, n, m)] u + F(i, n, m) c\right\}}{\frac{1}{m-1} \sum_{i=1}^{n} H(i, n, m) \min\{i, k\} [m - F(i, n, m)]}$$

where I have defined

$$H(i, n, m) \equiv \binom{n-1}{i-1} (1 - \frac{1}{m})^{n-i} \frac{1}{m}^{i-1} \frac{1}{i}$$

and $F(i, n, m) \equiv \frac{im-n}{m-1}$
Discussion of Lemma

- Naturally, $p^*(n, m; k)$ is increasing in $n$ and decreasing in $m$

- Moreover, $p^*(n, m; k)$ is strictly decreasing in $k$

- Given search frictions, low $k$ implies high probability of rationing

- Sellers have greater local monopoly power and charge higher price
Equilibrium Profits

PROPOSITION 1

a) If $k = n$, then $p^*(n, m; k) = c$ and, hence, $\pi^*(n, m; k) = 0$

b) If $k < n$, then $p^*(n, m; k) > c$ and, hence, $\pi^*(n, m; k) > 0$
Discussion of Proposition

- As long as there are *some* capacity constraints $p^* > c$ and $\pi^* > 0$

- Profits in the symmetric equilibrium are

$$
\pi^*(n, m; k) = \frac{n}{m} [p^*(n, m; k) - c] \sum_{i=1}^{n} H(i, n, m) \min\{i, k\}.
$$

- Equilibrium price, $p^*(n, m; k)$, is decreasing in $k$

- Expected number of matches, $\sum_{i=1}^{n} H(i, n, m) \min\{i, k\}$, is increasing in $k$

- Profit is hump-shaped, provided market tightness $n/m$ not too small
Equilibrium profit

Figure: Equilibrium profit for various $k$'s
A Model with Consumption Externalities

- Assume that \( k = n, \ p_i^j = p^j \)

- If seller \( j \) gets \( i \) customers, each enjoys \( u(i) \). For any \( h < i \), \( u(h) \geq u(i) \)

- Under this specification...

- Expected utility of a buyer who visits seller \( j \) is

\[
U^j \left( p^j, \theta \right) = \sum_{i=1}^{n} \binom{n-1}{i-1} (1 - \theta)^{n-i} \theta^{i-1} u(i) - p^j
\]

- The expected profit of seller \( j \) is

\[
\pi^j \left( p^j, \theta \right) = n\theta \left( p^j - c \right)
\]
The Seller’s Problem

- Once again, seller $j$ takes $\tilde{p}$ as given and chooses $p^j$ to

$$\max_{p^j} \pi^j(p^j, \theta)$$

- subject to the constraint

$$U^j(p^j, \theta) = U^l(\tilde{p}, \tilde{\theta}).$$

- where $\tilde{\theta} = (1 - \theta)/(m - 1)$
LEMMMA 2

In the unique symmetric equilibrium, every buyer visits each seller with probability $\theta^* = \frac{1}{m}$, and all sellers announce the price

$$p^* (n, m) = c + \frac{m}{m - 1} \sum_{i=1}^{n} H (i, n, m) i [1 - F (i, n, m)] u(i)$$

Like before,

$$H (i, n, m) \equiv \binom{n-1}{i-1} \left(1 - \frac{1}{m}\right)^{n-i} \left(\frac{1}{m}\right)^{i-1} \frac{1}{i}$$

and

$$F (i, n, m) \equiv \frac{im-n}{m-1}$$
Equilibrium Profits

PROPOSITION 2

a) If $u(i) = u$ for all $i$, then $p^*(n, m) = c$ and, therefore, $\pi^*(n, m) = 0$

b) If for all $i \in \{1, \ldots, n - 1\}$, $u(i) \geq u(i + 1)$, with strict inequality for some $i$, then $p^*(n, m) > c$ and, therefore, $\pi^*(n, m) > 0$. 
Example

- Let \( u(i) = ν - κ(i - 1) \), with \( κ > 0 \) and \( ν > c \).

- Under this specification, one can show that

\[
\begin{align*}
 p^*(n, m; κ) &= c + κ \frac{n - 1}{m - 1} \\
 π^*(n, m; κ) &= κ \frac{n(n - 1)}{m(m - 1)}
\end{align*}
\]

- Hence, we have

\[
\begin{align*}
 \frac{∂p^*(n, m; κ)}{∂κ} &> 0 \\
 \frac{∂π^*(n, m; κ)}{∂κ} &> 0
\end{align*}
\]
Intuition

- Large $\kappa$ means low utility from visiting a store with many customers.

- Buyers are willing to choose a seller with a higher price.

- This gives incentive to sellers to post higher prices.

- We still focus on symmetric equilibrium: all sellers post same price and get same number of expected buyers.

- However, the existence of congestion effects serves as a collusion device allowing sellers to boost equilibrium profits...

- ...even in the absence of any capacity constraints.
A Model with State Contingent Pricing

- Assume that \( k = n, \ u(i) = u > c \)

- Now seller \( j \) can advertise \( p^j = (p_1^j, \ldots, p_n^j) \)

- Examples: auctions, Groupon

- Expected utility of a buyer who visits seller \( j \) is

\[
U^j (p^j, \theta) = u - \sum_{i=1}^{n} \left( \begin{array}{c} n - 1 \\ i - 1 \end{array} \right) (1 - \theta)^{n-i} \theta^{i-1} p_i^j
\]

- The expected profit of seller \( j \) is

\[
\pi^j (p^j, \theta) = \sum_{i=1}^{n} \left( \begin{array}{c} n \\ i \end{array} \right) (1 - \theta)^{n-i} \theta^{i} i (p_i^j - c)
\]
LEMMMA 3
Every price vector \( p^* = (p_1^*, \ldots, p_n^*) \) that satisfies
\[
\frac{1}{m-1} \sum_{i=1}^{n} H(i, n, m) \ i \ [m - F(i, n, m)] \ p_i^* = c,
\]
\[
\frac{n}{m} \left[ \sum_{i=1}^{n} H(i, n, m) \ i \ p_i^* - c \right] \geq 0,
\]
and \( p_i^* \leq u \), for all \( i = \{1, \ldots, n\} \), together with a strategy for the buyers to visit each seller with probability \( \theta^* = 1/m \), constitutes a symmetric equilibrium.

- Equilibrium prices are not uniquely pinned down
- This result, first pointed out by Coles-Eeckhout (2003), is preserved in an environment with no capacity constraints
PROPOSITION 3

a) If sellers cannot apply state contingent pricing, then the unique symmetric equilibrium has \( p_i^* = c \) for all \( i = \{1, \ldots, n\} \) and \( \pi^*(n, m; p^*) = 0 \)

b) If sellers can apply state contingent pricing, a continuum of positive profit levels can be supported in symmetric equilibrium

c) In all symmetric equilibria, \( U^*(n, m; p^*) > 0 \)
Discussion of Proposition

- In the absence of state contingent pricing, \( p_i^* = c, \pi^* = 0 \)
- State-contingent pricing \( \Rightarrow \) continuum of not payoff equivalent equilibria
- When seller \( j \) can choose \( p_j^* \), she can advertise more buyer surplus in some states and less in other states. Other players are unaffected
- Intuitively, given any \( \tilde{p} \) by her rivals, seller \( j \) has a best response correspondence \( \Rightarrow \) multiplicity of equilibria (by symmetry)
- In small markets, various best responses lead to different levels of expected utility in the subgame \( \Rightarrow \) not unique sharing rule of surplus
- Sellers can never get the whole pie: suppose they could; this would require \( p_i^* = u \); but if \( p_i^* \) is fixed, it has to equal \( c \) leading to \( \pi^* = 0 \)
Here I do not provide a theory of refining equilibria.

\[ p_i^* = c \] is always an equilibrium, and it is preferred by buyers.

Sellers’ equilibrium profit can be written as

\[
\pi^* (n, m; p^*) = \frac{n}{m(m - 1)} \sum_{i=1}^{n} H(i, n, m) i [F(i, n, m) - 1] p_i^*
\]

Typically, high profit equilibria tend to assign a high weight (price) in states where \( i \) is large but not too close to \( n \).
Example

- The set of possible equilibria is very rich

- However, all one needs to support equilibria with $\pi^* > 0$ is *some* price differentiation among states

- Let $m = 2, n = 10$. Sellers can only choose $p^-$ (charged when 5 or less buyers show up) and $p^+$ (charged when 6 or more show up)

- Any $(p^-, p^+)$ that satisfies

\[
\frac{443}{256} p^- - \frac{187}{256} p^+ = c,
\]

\[
5 \left( \frac{1}{2} p^- + \frac{1}{2} p^+ - c \right) \geq 0,
\]

and $p^-, p^+ \leq u$, constitutes a symmetric equilibrium.
Example

Figure: Here $u = 1, c = 0.2$. All $p^-, p^+$ on the red line are consistent with equilibrium.
Comparison of the Three Environments

- Recall that with all three ingredients
  \[ U^j (p^j, \theta) = \sum_{i=1}^{n} \binom{n-1}{i-1} (1 - \theta)^{n-i} \theta^{i-1} \frac{\min\{i, k\}}{i} \left[ u(\min\{i, k\}) - p^j_i \right] \]

- If we remove
  - Capacity constraints \( \Rightarrow \min\{i, k\} = i \)
  - Congestion effects \( \Rightarrow u(\min\{i, k\}) = u \)
  - State contingent prices \( \Rightarrow p^j_i = p^j \)

- In a world without any of the three \( U^j (p^j, \theta) = u - p^j \)

- Sellers who announce prices higher than the competition get no customers.
  This leads to a price war that will end only when all sellers set \( p^* = c \)
Common Feature of the Three Ingredients

- Despite their different nature, the 3 ingredients share a common feature.

- They lead to an environment where customers exert **externalities** on one another, when at the same location.

- The externality could be affecting directly the utility, or the ability to consume, or even the price that buyers have to pay.

- In all three cases sellers win b/c of a **new economic force** introduced: buyers visit sellers with high prices hoping they will have few customers.

- Sellers accept the offer and post higher prices in equilibrium. In the game theory **jargon**: All three ingredients serve as **collusion** devices.

- Capacity constraints are just a **subcase** of a general market description where the BP fails to hold.
Welfare

- Measure of welfare: expected total surplus, $S^* \equiv nU^* + m\pi^*$

- In the three environments, we have

\[ S^*(n, m; k) = n(u - c) \sum_{i=1}^{n} H(i, n, m) \min\{i, k\}, \quad (1) \]

\[ S^*(n, m; u) = n \sum_{i=1}^{n} H(i, n, m) i [u(i) - c], \quad (2) \]

\[ S^*(n, m; p^*) = n(u - c). \quad (3) \]

- In (1), capacity constraints prevent matches from materializing

- In (2), the utility enjoyed depends on how many buyers show up

- In (3), surplus is independent of which equilibrium price prevails
Policy Implications?

- Modern societies enforce anti-trust laws trying to promote competition

- Main objective: prevent anticompetitive practices such as *price fixing*

- Here I have highlighted three collusion devices other than price fixing

- More importantly, these three alternative setups have different consequences on welfare
State Contingent Pricing

- Not much to say related to policy

- Different prices lead to different sharing rules of the same surplus

- Authorities should intervene only if they judge that the sharing rule of the surplus is unfair

- E.g. sellers are making excessive profits
Capacity Constraints

- $S^*$ is strictly increasing in $k$

- However, as we saw, $\exists! \nu < n$ s.t. $\pi^*(n, m; k)$ is decreasing in $k$, for $k \geq \nu$

- If sellers could fix capacities they would not choose the socially optimal $k$

- Example:
  - $m = 5, n = 50, u = 1,$ and $c = 0.2$
  - Sellers can silently agree on a value of $k$
    - Then, given that $k$, they legitimately compete over prices

- Sellers would set $k = 7$ leading to $\pi^*(7) = 4.692$ and $S^*(7) = 27.29$

- If the authorities could enforce $k = 8$, $S^*(8) = 30.53$, but $\pi^*(8) = 4.690$

- $S^*(9) = 33.3$, $\pi^*(9) = 4.35$; $S^*(10) = 35.3$, $\pi^*(10) = 3.72$;
  $S^*(13) = 39.1$, $\pi^*(13) = 1.38$
Congestion Effects

- Again assume \( u(i) = v - \kappa(i - 1) \), with \( \kappa > 0, v > c \)

- Under this specification, one can show that

\[
S^*(n, m; u) = \frac{n [m(v - c) - \kappa(n - 1)]}{m} \Rightarrow \frac{\partial S^*(n, m; u)}{\partial \kappa} < 0
\]

- But, as we saw, \( \frac{\partial \pi^*}{\partial \kappa} > 0 \)

- A conflict arises: sellers want big \( \kappa \); social optimality requires \( \kappa = 0 \)

- Authorities should be concerned about this conflict, since sellers have an incentive to agree on practices that increase \( \kappa \) and decrease \( S^* \) artificially

- More importantly, these practices do not violate any antitrust laws
Conclusions

- I revisit the BP through the lens of a directed search model
- I augment the baseline model with 1) capacity constraints, 2) consumption externalities, and 3) state contingent pricing, in isolation
- In all three cases, buyers exert a negative externality on one another
- Buyers dislike stores with many customers and, thus, are willing to visit sellers with higher prices
- In equilibrium, sellers do precisely that: they announce higher prices
- The directed search model offers a new perspective of looking at the BP: Existence of capacity constraint is just a subcase of a more general market description where the Paradox fails to hold