Production, Distribution and Allocation
Roadmap to this Lecture

1. The economy in 3 markets and 3 agents
2. Properties of a production function:
   1. Output is always positive
   2. More inputs more output
   3. Constant returns to scale
   4. Diminishing marginal returns
3. Determining factor demand and factor prices
4. Income distribution
Overview:

- We are interested in describing the production side of the economy
- How are the prices of the factors of production determined?
- How is total income distributed?
The Economy in the Long-Run

- **Basic Assumption:** in the long-run, prices are fully flexible and markets clear.

- This assumption is a basic assumption of the Classical Paradigm.
A Macroeconomy in 3 Markets

- **The Goods and Services Market:** where goods and services are traded
- **The Factors of Production Market:** where labor is hired and capital is rented
- **Financial Market:** where household savings are channeled into investment
A Macroeconomy in 3 Agents

- **Households**: consume goods and services (demand) and provide labor (supply) and savings (supply).
- **Firms**: produced goods and services (supply), hire labor and capital (demand), and invest (demand)
- **Government**: provides goods and services (supply) and also consumes them (demand). It also borrows (demand)
## A Flow Matrix

<table>
<thead>
<tr>
<th></th>
<th>Households</th>
<th>Government</th>
<th>Firms</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Goods and Services</strong></td>
<td>Consumption [Demand]</td>
<td>Govt. Expenditures [Demand]</td>
<td>Investment [Demand]</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Public Good [Supply]</td>
<td></td>
</tr>
<tr>
<td><strong>Labor</strong></td>
<td>Work [Supply]</td>
<td>Public Employees [Demand]</td>
<td>Employees [Demand]</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Financial</strong></td>
<td>Savings [Supply]</td>
<td>Deficit Financing [Demand]</td>
<td>Investment [Demand]</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

ECN 101 - MACROECONOMICS
Factors of production

\[ K = \text{capital, tools, machines, and structures used in production} \]

\[ L = \text{labor, the physical and mental efforts of workers} \]
The production function

- The Production function: \( Y = F(K, L) \)
- shows how much output (\( Y \)) the economy can produce from \( K \) units of capital and \( L \) units of labor.
- reflects the economy’s level of technology.
Returns to scale: a review

Initially \( Y_1 = F(K_1, L_1) \)

Scale all inputs by the same factor \( z \):

\[
K_2 = zK_1 \quad \text{and} \quad L_2 = zL_1
\]

(If \( z = 1.25 \), then all inputs are increased by 25%)

What happens to output, \( Y_2 = F(K_2, L_2) \)?

- If *constant returns to scale*, \( Y_2 = zY_1 \)
- If *increasing returns to scale*, \( Y_2 > zY_1 \)
- If *decreasing returns to scale*, \( Y_2 < zY_1 \)
The Cobb-Douglas Production Function

\[ F(K, L) = AK^\alpha L^\beta \]

\[ A, \alpha, \beta > 0 \]
Properties of Production Functions
1. Output is always Positive

- Mathematically:

\[ F(K, L) \geq 0 \text{ for any } K, L \geq 0 \]
Marginal product of labor (MPL)

def:
The extra output the firm can produce using an additional unit of labor (holding other inputs fixed):

\[ MPL = F(K, L + 1) - F(K, L) \]
Exercise: *compute & graph MPL*

a. Determine \( MPL \) at each value of \( L \)

b. Graph the production function

c. Graph the \( MPL \) curve with \( MPL \) on the vertical axis and \( L \) on the horizontal axis

<table>
<thead>
<tr>
<th>( L )</th>
<th>( Y )</th>
<th>( MPL )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>n.a.</td>
</tr>
<tr>
<td>1</td>
<td>10</td>
<td>?</td>
</tr>
<tr>
<td>2</td>
<td>19</td>
<td>?</td>
</tr>
<tr>
<td>3</td>
<td>27</td>
<td>8</td>
</tr>
<tr>
<td>4</td>
<td>34</td>
<td>?</td>
</tr>
<tr>
<td>5</td>
<td>40</td>
<td>?</td>
</tr>
<tr>
<td>6</td>
<td>45</td>
<td>?</td>
</tr>
<tr>
<td>7</td>
<td>49</td>
<td>?</td>
</tr>
<tr>
<td>8</td>
<td>52</td>
<td>?</td>
</tr>
<tr>
<td>9</td>
<td>54</td>
<td>?</td>
</tr>
<tr>
<td>10</td>
<td>55</td>
<td>?</td>
</tr>
</tbody>
</table>
Production function

Output (Y) vs Labor (L)

Marginal Product of Labor

MPL (units of output) vs Labor (L)

 answers:
2. More inputs, more output

- one extra unit of input (either L or K), everything else equal, delivers at worst nothing, at best, more output.

- Mathematically:
  - $F_K = \frac{\partial F}{\partial K} \geq 0$, marginal productivity of capital: How much more output if I increase K by 1 unit.
  - $F_L = \frac{\partial F}{\partial L} \geq 0$, marginal productivity of labor: How much more output if I increase labor by 1 unit.
Partial Derivatives

- **Example:** the Cobb-Douglas production function

\[ F(K,L) = AK^\alpha L^\beta \]

- \[ F_K = A(\alpha K^{\alpha-1}) L^\beta = \alpha Y/K \geq 0 \]

- \[ F_L = AK^\alpha (\beta L^{\beta-1}) = \beta Y/L \geq 0 \]
3. Constant Returns to Scale

- Cobb-Douglas production function:
  \[ Y = AK^\alpha L^\beta \]
  \[ Y^* = A(zK)^\alpha (zL)^\beta = Yz^{(\alpha + \beta)} \]

Hence:
- If \( \alpha + \beta > 1 \) then \( Y^* > F(zK, zL) \), IRS
- If \( \alpha + \beta = 1 \) then \( Y^* = F(zK, zL) \), CRS
- If \( \alpha + \beta < 1 \) then \( Y^* < F(zK, zL) \), DRS
4. Diminishing Marginal Returns

- The marginal productivity of an extra unit of an input declines the more of that input is used in production

- Mathematically:
  
  \(- F_{KK} \leq 0\)
  \(- F_{LL} \leq 0\)
The MPL and the production function

As more labor is added, \( MPL \downarrow \)

Slope of the production function equals \( MPL \)

Output \( Y \)

Labor \( L \)
**Diminishing marginal returns**

- As a factor input is increased, its marginal product falls (other things equal).
- Intuition:
  \[ \uparrow L \text{ while holding } K \text{ fixed} \]
  \[ \Rightarrow \text{fewer machines per worker} \]
  \[ \Rightarrow \text{lower productivity} \]
5. Determinants of Output Growth

\[ Y \text{ growth} = (MPK \times K \text{ share}) \times K \text{ growth} + (MPL \times L \text{ share}) \times L \text{ growth} \]

Mathematically: \( Y = F(K,L) \)

\[ dY = F_K \, dK + F_L \, dL \text{ hence} \]

\[ \frac{dY}{Y} = \left( \frac{F_K K}{Y} \right) \frac{dK}{K} + \left( \frac{F_L L}{Y} \right) \frac{dL}{L} \]
Cobb-Douglas Example

F_K = \alpha Y/K; and F_L = \beta Y/L. Hence:

\[ \frac{dY}{Y} = \left( \alpha \frac{Y}{K} \right) \frac{dK}{K} + \left( \beta \frac{Y}{L} \right) \frac{dL}{L} \]

or simply

\[ \frac{dY}{Y} = \alpha \frac{dK}{K} + \beta \frac{dL}{L} \]

\(\alpha\) is the capital share; \(\beta\) is the labor share
Factor Prices and Quantities of Equilibrium
Assumptions of the model

1. Technology is fixed. $A = \bar{A}$
2. The economy’s supplies of capital and labor are fixed at $K = \bar{K}$ and $L = \bar{L}$
Determining GDP

Output is determined by the fixed factor supplies and the fixed state of technology:

$$\bar{Y} = F(\bar{K}, \bar{L})$$
The distribution of national income

- determined by factor prices, the prices per unit that firms pay for the factors of production.

- The wage is the price of $L$, the rental rate is the price of $K$. 
**Notation**

\[ W = \text{nominal wage} \]
\[ R = \text{nominal rental rate} \]
\[ P = \text{price of output} \]
\[ \frac{W}{P} = \text{real wage} \]
\[ \text{(measured in units of output)} \]
\[ \frac{R}{P} = \text{real rental rate} \]
What Real Wage Really Means

- $W$ is measured in $/\text{unit of work}$
- $P$ is measured in $/\text{unit of good}$

Hence

$$\frac{W}{P} = \frac{($/\text{unit of work})}{($/\text{unit of good})} = \frac{\text{units of good}}{\text{units of labor}}$$
How factor prices are determined

- Factor prices are determined by supply and demand in factor markets.
- Recall: Supply of each factor is fixed.
- What about demand?
Survey: Grads finding hot jobs market

Employers set to hire nearly 15% more college grads this spring, survey says; starting salaries up for many business, engineering majors.

[...]accounting degree graduates are receiving an average starting salary of $46,188, up 5.4 percent from a year ago. Right behind are economics/finance graduates, who are getting average offers of $45,058, up 5.3 percent, and business administration/management majors, who are seeing average offers 3.9 percent higher than a year ago at $40,976.

Source: CNNMoney.com
Demand for labor

- Assume markets are competitive: each firm takes \( W, R, \) and \( P \) as given.

- Basic idea: A firm hires one extra unit of labor if the cost does not exceed the benefit.
  
  cost = real wage \((W/P)\)
  
  benefit = marginal product of labor \((MPL)\)
Check your understanding:

Which of these production functions have diminishing marginal returns to labor?

a) \[ F(K, L) = 2K + 15L \]

b) \[ F(K, L) = \sqrt{KL} \]

c) \[ F(K, L) = 2\sqrt{K} + 15\sqrt{L} \]
Exercise (part 2)

Suppose \( W/P = 6 \).

\[ L \quad Y \quad MPL \]
\[ 0 \quad 0 \quad \text{n.a.} \]
\[ 1 \quad 10 \quad 10 \]
\[ 2 \quad 19 \quad 9 \]
\[ 3 \quad 27 \quad 8 \]
\[ 4 \quad 34 \quad 7 \]
\[ 5 \quad 40 \quad 6 \]
\[ 6 \quad 45 \quad 5 \]
\[ 7 \quad 49 \quad 4 \]
\[ 8 \quad 52 \quad 3 \]
\[ 9 \quad 54 \quad 2 \]
\[ 10 \quad 55 \quad 1 \]

d. If \( L = 3 \), should firm hire more or less labor? Why?

e. If \( L = 7 \), should firm hire more or less labor? Why?
Each firm hires labor up to the point where \( MPL = \frac{W}{P} \).
The equilibrium real wage

Units of output

Labor supply

MPL, Labor demand

\( \bar{L} \)

Units of labor, \( L \)

The real wage adjusts to equate labor demand with supply.
Determining the rental rate

We have just seen that \( MPL = \frac{W}{P} \)

The same logic shows that \( MPK = \frac{R}{P} \):

- diminishing returns to capital: \( MPK \downarrow \) as \( K \uparrow \)
- The \( MPK \) curve is the firm’s demand curve for renting capital.
- Firms maximize profits by choosing \( K \) such that \( MPK = \frac{R}{P} \).
The equilibrium real rental rate adjusts to equate demand for capital with supply.

Units of output

Supply of capital

equilibrium \( R/P \)

MPK, demand for capital

Units of capital, \( K \)

The real rental rate
The Neoclassical Theory of Distribution

- states that each factor input is paid its marginal product
- accepted by most economists
How income is distributed:

- total labor income $= \frac{W}{P} \bar{L} = MPL \times \bar{L}$
- total capital income $= \frac{R}{P} \bar{K} = MPK \times \bar{K}$

If production function has constant returns to scale, then

$$\bar{Y} = MPL \times \bar{L} + MPK \times \bar{K}$$

- national income
- labor income
- capital income
Euler’s Theorem and Income Distribution

- Euler’s theorem states that if $F(X,Y)$ is a homogenous function of degree one such that $F(zX, zY) = zF(X,Y)$. Then:

$$F(X,Y) = F_X X + F_Y Y$$

- An implication of Euler’s theorem is that for any production function with CRS, then

$$F(K,L) = F_K K + F_L L$$
A Cobb-Douglas Example

- When $\alpha + \beta = 1$, we know that the Cobb-Douglas production function has CRS. Hence, a direct application of Euler’s theorem or direct application of the economic arguments presented above:

$$\text{MPK} \times K + \text{MPL} \times L = \alpha Y + (1-\alpha)Y = Y$$
Recap: Assumptions Made

- **CRS technology**: even when some industries seem to violate this assumption, it holds true for entire economies fairly well.

- **Perfect Competition**: is a good approximation to long-run behavior even if it is violated often.

- **Profit Maximization**: hardly needs justification...
Outline of model

A *closed economy, market-clearing model*

**Supply side**
- factor markets (supply, demand, price)
- determination of output/income

**Demand side**

**Next**
- determinants of $C$, $I$, and $G$

**Equilibrium**
- goods market
- loanable funds market