The Response of Term Rates to Monetary Policy Uncertainty

Features of the Model

• Typical Limited Participation set-up

• Monetary policy modeled as a Markov process for the monetary growth rate so as to obtain exact solutions rather than linearizations.

• Properties:
  1. Model displays liquidity effect.
  2. Household saving decisions are done before the state of the world is revealed; hence increased uncertainty affects these decisions.
  3. Nominal interest rates respond to Fisherian and liquidity factors.
Timing of Events in Limited Participation Model

- Households send $I_t$ each to the financial intermediary.
- Financial intermediary borrows $W_t N_t$ from the financial intermediary at a gross interest rate $R_t$.
- Intermediate goods firms repay $R_t W_t N_t$ to the financial intermediary and households receive profits and interest on their deposits at rate $R_t$.
- Money growth rate $\gamma$ and technology shock $z_t$ are realized.
- Production occurs and the goods market closes.

Flow of Funds in Model

Central Bank

$\gamma_t$

$I_t$

$W_t N_t$

$R_t (I_t + \gamma_t)$

$R_t W_t N_t$

Households

Financial Intermediary

Firms

$D_t + r_t K_t$

$R_t C_t$
Necessary Conditions

- **Firm’s Labor Demand:**
  \[ R_t \frac{W_t}{P_t} = (1 - \alpha)h_t^{-\alpha} \]

- **Household’s Optimum:**
  \[ E_{t-1} \left[ \frac{U_{c,t}}{P_t} - R_t \beta E_t \left( \frac{U_{c,t+1}}{P_{t+1}} \right) \right] = 0 \]
  \[ U_{l,t} = U_{c,t} \frac{W_t}{P_t} \]

- **Central Bank:**
  \[ \bar{M}_t = \bar{M}_{t-1}(1 + g_t) \]
The Money Growth Process:

\[ g_t = \begin{cases} 
  g_1 = \tilde{g} - \delta \\
  g_2 = \tilde{g} \\
  g_3 = \tilde{g} \\
  g_4 = \tilde{g} + \delta 
\end{cases} \]

\[ \Pi = \begin{bmatrix}
  \pi & \frac{1-\pi}{3} & \frac{1-\pi}{3} & \frac{1-\pi}{3} \\
  \frac{1-\pi}{3} & \pi & \frac{1-\pi}{3} & \frac{1-\pi}{3} \\
  \frac{1-\pi}{2} & 0 & \pi & \frac{1-\pi}{2} \\
  \frac{1-\pi}{3} & \frac{1-\pi}{3} & \frac{1-\pi}{3} & \pi 
\end{bmatrix} \]

| \( s_i \) | \( E(g_{t+1}|s_t = s_i) \) | \( Var(g_{t+1}|s_t = s_i) \) |
|---|---|---|
| 1 | \( \tilde{g} - \delta \frac{(4\pi - 1)}{3} \) | \( \frac{2}{9} \left( 1 + 7\pi + 8\pi^2 \right) \delta^2 \) |
| 2 | \( \tilde{g} \) | \( \frac{2}{3} (1 - \pi) \delta^2 \) |
| 3 | \( \tilde{g} \) | \( (1 - \pi) \delta^2 \) |
| 4 | \( \tilde{g} + \delta \frac{(4\pi - 1)}{3} \) | \( \frac{2}{9} \left( 1 + 7\pi + 8\pi^2 \right) \delta^2 \) |
Equilibrium

Goods Market:

\[ c_t = h_t^{1-\alpha} \]

Loan Market:

\[ T_t + I_t = L_t = W_t h_t \]

Labor Market: assuming \( U(c_t, 1-h_t) = \frac{c_t^{1-\gamma}}{1-\gamma} + A(1-h_t) \), then

\[ Ac_t^\gamma = \frac{W_t}{P_t} = (1-\alpha) \frac{h_t^{-\alpha}}{R_t} \]
Equilibrium (cont.)

- In a stationary equilibrium, all nominal quantities are scaled by the beginning of the period money stock.
- Equilibrium is characterized by 36 values \((h_{ij}, R_{ij}, i_i)\)
Introducing other Assets

• $\rho_t$ a one period real bond (in units of consumption good)
• $R_t^I$ a one period nominal bond
• $R_t^{II}$ a two period nominal bond

These bonds trade in a market that is open after money growth is observed.
The difference between $R_t$ and $R_t^I$ is that in the former, uncertainty has not yet been resolved.
Pricing formulas from usual asset pricing:

\[ U_{c,t} = \rho_t \beta E_t \left[ U_{c,t+1} \right] \]

\[ \frac{U_{c,t}}{P_t} = R_t^I \beta E_t \left( \frac{U_{c,t+1}}{P_{t+1}} \right) \]

\[ \frac{U_{c,t}}{P_t} = (R_t^{II})^2 \beta E_t \left( \frac{U_{c,t+1}}{P_{t+1}} \frac{1}{R_{t+1}^I} \right) \]
Equilibrium Characteristics

<table>
<thead>
<tr>
<th>Variable</th>
<th>Low Variance (s_t = s_{t-1} = s_2)</th>
<th>High Variance (s_t = s_{t-1} = s_3)</th>
<th>Low Variance (s_t = s_{t-1} = s_2)</th>
<th>High Variance (s_t = s_{t-1} = s_3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>i_{t-1}</td>
<td>58.0</td>
<td>58.4</td>
<td>57.1</td>
<td>57.3</td>
</tr>
<tr>
<td>h_{it-1}</td>
<td>40.2</td>
<td>40.3</td>
<td>40.0</td>
<td>40.1</td>
</tr>
<tr>
<td>R_{it-1} - 1</td>
<td>7.3</td>
<td>6.6</td>
<td>8.8</td>
<td>8.5</td>
</tr>
<tr>
<td>R'^{it-1} - 1</td>
<td>8.2</td>
<td>7.8</td>
<td>9.1</td>
<td>8.9</td>
</tr>
<tr>
<td>TP_{it-1}</td>
<td>1.3</td>
<td>1.6</td>
<td>0.50</td>
<td>0.70</td>
</tr>
</tbody>
</table>

Greater uncertainty lowers certainty equivalence of next period consumption – This causes real interest rates to fall. Also, deposits to banking sector increase. This causes additional fall in nominal rates (note effects of expected inflation.) Output increases slightly (more labor) and term premium increases. So yield curve falls and steepens.
Measuring the Effects of Time Varying Uncertainty

Tools we have

- VARS: useful in measuring the dynamic responses of the conditional mean of a system of variables

- Uncertainty is usually associated with variance: one model for time-varying variance is the GARCH model

Looks like we should combine the two!

A typical VAR

\[ Y = X\Pi + u \]
\[ E(u'u) = \Omega \]
\[ \Omega = A'DA \]

\[ YA^{-1} = X\Pi A^{-1} + \epsilon \]
A Multivariate GARCH for the $\epsilon$

$$H_t i = W + \Gamma H_{t-1} i + \Phi \epsilon_{t-1}^2 \quad \text{GARCH}(1,1)$$

$H_t$ is an $n \times n$ matrix of the conditional variances of the $\epsilon$

$i$ is an $n \times 1$ vector of ones

Notice: because the $\epsilon$ are the structural and orthogonalized errors of the VAR, we need not worry about the conditional covariances – the Achilles heel of multivariate ARCH models.

Combining the VAR with the multivariate GARCH: The GARCH-SVAR

$$YA^{-1} = X\Pi A^{-1} + H\beta A^{-1} + \epsilon$$
Maximum Likelihood Estimation – An Iterative Procedure

\[
L(\theta) = -\frac{n(T - p)}{2} \log(2\pi) - \frac{1}{2} \sum_{t=2}^{T} \left( \log |H_t| + \epsilon_t' H_t^{-1} \epsilon_t \right)
\]

Iterations

1. Estimate usual VAR and use Choleski to obtain \( \hat{\epsilon}^0 \)
2. Estimate GARCH model and obtain \( \hat{H}^0 \)
3. Estimate GARCH-SVAR and obtain \( \hat{\epsilon}^1 \)
4. Estimate GARCH model and obtain \( \hat{H}^1 \)
5. Iterate on steps 3 and 4 until convergence.
Impulse Response Functions and the Effects of Time-Varying Uncertainty

\[
\frac{\Delta y_{j,t+s}}{\Delta \varepsilon_{i,t}} = E(y_{j,t+s} \mid \varepsilon_{it} = \varepsilon^*_i; y_{t-1}, H_t; y_{t-2}, H_{t-1}; ...) - E(y_{j,t+s} \mid \varepsilon_{it} = \varepsilon^*_i; y_{t-1}, H_t = 0; y_{t-2}, H_{t-1} = 0; ...) +
E(y_{j,t+s} \mid \varepsilon_{it} = \varepsilon^*_i; y_{t-1}; H_t = 0; y_{t-2}, H_{t-1} = 0; ...) - E(y_{j,t+s} \mid \varepsilon_{it} = 0; y_{t-1}; H_t; y_{t-2}, H_{t-1}; ...)
\]

Effects of time-varying uncertainty: calculate the IRFs and compare to setting the \( \beta = 0 \)

Properties:

1. Shock to the \( \varepsilon \) affects the conditional mean via two channels: (a) direct conditional mean, and (2) conditional variance channel.
2. IRFs are no longer symmetric: the sign of the shock matters since the conditional variance effect is always positive.
3. IRFs are no longer scale invariant: the size of the shock matters.

Variables: \( EM, P, PCOM, FF, NBRX, \Delta M2, TB3, TB6 \)

Identification: Choleski given the ordering

Figure 1 – Comparing typical IRFs with GARCH-SVAR IRFs

Figure 2 – Conditional Variance Estimates

Figure 3 – The marginal effect of time-varying uncertainty
Figure 1 – Comparing a Typical SVAR with the GARCH-SVAR

Notes: Responses to a 0.5% shock in the federal funds rate equation. All plots measured in percentages. Thick solid lines are the responses from the GARCH-SVAR, thin solid lines are the responses from a typical SVAR, and dotted lines are the one-standard deviation error bands obtained by Monte Carlo simulation.
Figure 2 – Conditional Variance Estimates from GARCH-SVAR Model

Notes: Conditional variances calculated from the GARCH-SVAR model.
Figure 3 – Measuring Volatility and Asymmetry in the GARCH-SVAR

Notes: Thick solid line represents responses to a -3% shock in the federal funds rate equation; thin solid line represents the inverted responses to a +3% shock instead; and the dotted line represents the responses to a -3% shock when the volatility effects on the conditional mean are set to zero.
Conclusions

- A clever solution to a limited participation model with second moment effects:
  - Term rates inversely related to uncertainty
  - In the short-term, increased uncertainty increases liquidity. In the long-run, the convexity of money growth modifies the certainty equivalence of a dollar in the future.

- A clever empirical model: the GARCH-SVAR

- Future directions:
  - GARCH-SVAR provides direct measures of inflation and output volatility that can be included in the policy equation.
  - Identification strategy: find the space of linear combinations that restricts GARCH effects to the diagonal