FEDERAL RESERVE INTEREST RATE TARGETING, RATIONAL EXPECTATIONS AND THE TERM STRUCTURE

BY

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The Predictive Ability of the Term Structure

Let $r(1)_t$ and $r(2)_t$ be the yields on one and two period (zero-coupon) bonds respectively.

Rational Expectations Hypothesis with a term premium:

$$r(2)_t = \frac{1}{2} [r(1)_t + E_t r(1)_{t+1}] + \pi(2)$$

Realizing that

$$r(1)_{t+1} = E_t r(1)_{t+1} + \varepsilon_{t+1}$$
then we can combine these equations into

$$\frac{1}{2} [r(1)_{t+1} - r(1)_t] = \alpha + \beta [r(2)_t - r(1)_t] + \nu_{t+1}$$

Rational Expectations suggests that $\beta = 1; \alpha = -\pi(2)$
Table 1
Estimates of the predictive power of the spread between one-period and two-period rates at various maturities

<table>
<thead>
<tr>
<th>Maturity of one-period rate</th>
<th>$\hat{\beta}$</th>
<th>s.e.$(\hat{\beta})$</th>
<th>tstat$(\hat{\beta})$</th>
<th>Source; table number; sample period</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 weeks</td>
<td>0.61*</td>
<td>0.15</td>
<td>4.12</td>
<td>Hardouvelis, 1988; 2; 1972–79</td>
</tr>
<tr>
<td>2 weeks</td>
<td>0.85*</td>
<td>0.09</td>
<td>9.63</td>
<td>Hardouvelis, 1988; 2; 1979–82</td>
</tr>
<tr>
<td>2 weeks</td>
<td>0.76*</td>
<td>0.05</td>
<td>15.55</td>
<td>Hardouvelis, 1988; 2; 1982–85</td>
</tr>
<tr>
<td>1 month</td>
<td>0.50*</td>
<td>0.12</td>
<td>4.21</td>
<td>Campbell and Shiller, 1991; 2; 1952–87</td>
</tr>
<tr>
<td>1 month</td>
<td>0.46*</td>
<td>0.07</td>
<td>6.57</td>
<td>Fama, 1984; 4; 1959–82</td>
</tr>
<tr>
<td>1 month</td>
<td>0.69*</td>
<td>0.17</td>
<td>4.06</td>
<td>Mishkin, 1988; 1; 1974–79</td>
</tr>
<tr>
<td>1 month</td>
<td>0.40*</td>
<td>0.08</td>
<td>4.44</td>
<td>Mishkin, 1988; 1; 1959–86</td>
</tr>
<tr>
<td>2 months</td>
<td>0.20</td>
<td>0.28</td>
<td>0.69</td>
<td>Campbell and Shiller, 1991; 2; 1952–87</td>
</tr>
<tr>
<td>3 months</td>
<td>−0.15</td>
<td>0.20</td>
<td>−0.74</td>
<td>Campbell and Shiller, 1991; 2; 1952–87</td>
</tr>
<tr>
<td>3 months</td>
<td>0.08</td>
<td>0.24</td>
<td>0.25</td>
<td>Froot, 1989; 1; 1969–86</td>
</tr>
<tr>
<td>3 months</td>
<td>0.23</td>
<td>0.19</td>
<td>1.24</td>
<td>Mankiw and Miron, 1986; 1; 1959–79</td>
</tr>
<tr>
<td>3 months</td>
<td>0.44</td>
<td>0.37</td>
<td>1.17</td>
<td>Robert et al., 1994; 8; 1975–79</td>
</tr>
<tr>
<td>3 months</td>
<td>−0.01</td>
<td>0.03</td>
<td>−0.29</td>
<td>Robert et al., 1994; 8; 1984–91</td>
</tr>
<tr>
<td>3 months</td>
<td>−0.21</td>
<td>0.26</td>
<td>−0.81</td>
<td>Shiller et al., 1983; 3; 1959–82</td>
</tr>
<tr>
<td>3 months</td>
<td>0.29</td>
<td>0.16</td>
<td>1.83</td>
<td>Shiller et al., 1983; 3; 1959–79</td>
</tr>
<tr>
<td>3 months</td>
<td>0.02</td>
<td>0.22</td>
<td>0.09</td>
<td>Simon, 1989; 1; 1961–88</td>
</tr>
<tr>
<td>6 months</td>
<td>0.04</td>
<td>0.33</td>
<td>0.13</td>
<td>Campbell and Shiller, 1991; 2; 1952–87</td>
</tr>
<tr>
<td>1 year</td>
<td>−0.13</td>
<td>0.27</td>
<td>−0.48</td>
<td>Blough, 1994; 3; 1949–89</td>
</tr>
<tr>
<td>1 year</td>
<td>−0.02</td>
<td>0.37</td>
<td>−0.05</td>
<td>Campbell and Shiller, 1991; 2; 1952–87</td>
</tr>
<tr>
<td>1 year</td>
<td>0.38</td>
<td>0.27</td>
<td>1.41</td>
<td>Cook and Hahn, 1990; 3; 1952–83</td>
</tr>
<tr>
<td>1 year</td>
<td>0.09</td>
<td>0.28</td>
<td>0.32</td>
<td>Fama and Bliss, 1987; 3; 1965–85</td>
</tr>
<tr>
<td>2 years</td>
<td>0.14</td>
<td>0.62</td>
<td>0.22</td>
<td>Campbell and Shiller, 1991; 2; 1952–87</td>
</tr>
<tr>
<td>5 years</td>
<td>2.79*</td>
<td>0.96</td>
<td>2.90</td>
<td>Campbell and Shiller, 1991; 2; 1952–87</td>
</tr>
</tbody>
</table>

The tstat$(\hat{\beta})$ entries are t-statistics for the hypothesis that $\hat{\beta} = 0$ in Eq. (3). All rates are based on discount or zero-coupon yields for U.S. Treasury securities.

*Significant at the 1 percent level.
Overnight vs. Monthly Yields

\( r_t \): overnight rate

\( r(n)_t \): \( n \)-period rate

\( \pi(n) \): constant premium

Rational Expectations suggests,

\[
r(n)_t = \frac{1}{n} \left[ r_t + \mathbb{E}_t \sum_{i=1}^{n-1} r_{t+i} \right] + \pi(n)
\]
Rearranging

\[
\frac{1}{n} \left[ \sum_{i=0}^{n-1} r_{t+i} \right] - r_t = \alpha + \beta [r(n)_t - r_t] + \nu_{t+n-1}
\]

with

\[
\alpha = -n \pi(n); \beta = 1; \nu_{t+n-1} = \frac{1}{n} \sum_{i=0}^{n} \varepsilon_{t+i}
\]
Summary of Results

- **TS1 – Overnight Spreads**: spreads between the overnight fed funds and the 1- and 3-month rates are good predictors of the change from the current daily rate to the average daily rate that will prevail over the next 1 to 3 months.

- **TS2 – Short-Term Spreads**: spreads between the very short-term bills, 30-60 days, are able to predict future changes in short-rates at horizons of not more than one month.
• TS3 – Medium-Term Spreads: spreads involving bills of maturities between 3 and 12 months have essentially no predictive power.

• TS4 – Long-Term Spreads: maturities over 1-2 years have some predictive content.
Fig. 1. Estimates of the predictive power of the spread between one-period and two-period rates at various maturities.
Federal Reserve Targeting

Transitory Deviations from Target

\[
    r_t = \bar{r}_t + u_t
\]
\[
    u_t = 0.017 + 0.384u_{t-1} + e_t
\]

Daily deviations from target are small (17 basis points) and short-lived, virtually disappearing in three days: \(0.384^3 = 0.06\).
The Size of Target Changes

Let $\delta_t = \bar{r}_t - \bar{r}_{t-1}$, which is zero for most days of the sample, otherwise, $\delta_t \in \{-1/8, 1/4, 3/8, 1/2\}$.

$$
\delta_t = \begin{cases} 
\eta & \text{with } P_t^+ \\
0 & \text{with } 1 - P_t^+ - P_t^- \\
-\eta & \text{with } P_t^-
\end{cases}
$$

$\eta > 0; \eta \sim f(\eta)$, the empirical distribution function.
Timing of Target Changes

Figure 2 suggests it is unlikely that $P_t^+ = P_t^-$. Rather, there are usually many steps in the same direction.

$$P_t^+ = \begin{cases} P^{--} & \text{if } \delta_{t-\tau} < 0, \\ P^{++} & \text{if } \delta_{t-\tau} > 0 \end{cases}, \quad P_t^- = \begin{cases} P^{--} & \text{if } \delta_{t-\tau} < 0 \\ P^{--} & \text{if } \delta_{t-\tau} > 0 \end{cases}$$

Formal statistical tests suggest that

$$P^{--} = P^{++} = P^s$$

but $P^s \neq P^d$
Additional finding: there seems to be little duration dependence beyond a 25 day horizon.
Summary of Properties

FR1: Daily deviations from target are transitory
FR2: There is interest rate smoothing – the target is adjusted in limited amounts in the same direction.
FR3: There is medium-term persistence – the target is set at a level that the Fed expects to maintain.
FR4: The Fed’s long-run objectives are price and output stability.
Reconciling Fed Behavior and Term Structure Evidence

**Approach:** generate synthetic data that takes into account the peculiar way the Fed targets interest rates and imposes the REH on the term structure. Do we get results that mirror those found in the literature?

**Ingredients of the Simulation**

1. 

\[ r_t = \bar{r}_t + u_t \]
\[ u_t = 0.017 + 0.384u_{t-1} + \hat{e}_t \]
where the $\hat{e}_t$ are bootstrapped from the original regression.

2.

$$\tilde{r}_t = \tilde{r}_{t-1} + \delta_t$$

$$\delta_t = \begin{cases} 
\eta & \text{with } P_t^+ \\
0 & \text{with } 1 - P_t^+ - P_t^- \\
-\eta & \text{with } P_t^- 
\end{cases}$$

$\eta$ is determined from the historical observed frequencies
3. Timing of target changes

\[ P_t^+ = \begin{cases} 
P^d & \text{if } \delta_{t-\tau} < 0 \\
P^s & \text{if } \delta_{t-\tau} > 0 
\end{cases} \quad P_t^- = \begin{cases} 
P^s & \text{if } \delta_{t-\tau} < 0 \\
P^d & \text{if } \delta_{t-\tau} > 0 
\end{cases} \]

with \( P^s = P^d = 0.020 \) for \( \tau \geq 25 \)

4. Term structure

\[ r(n)_t = \frac{1}{n} \left[ r_t + E_t \sum_{i=1}^{n-1} r_{t+i} \right] \]
where expectations are based on 1-3.

**Step 1:** generate the target data and the $r_t$.

**Step 2:** generate the entire term structure based on 4.

**Step 3:** Estimate 3 regressions

$r_t, r(28)_t$ 1-month, $r(56)_t$ 3-months, $r(84)_t$ 3-months, $r(168)_t$ 6-months
Term structure regressions of:

1. $r_t$ on $r(84)_t$ to obtain $\hat{\beta}_{TS1}$

2. $r(28)_t$ on $r(56)_t$ to obtain $\hat{\beta}_{TS2}$

3. $r(84)_t$ on $r(168)_t$ to obtain $\hat{\beta}_{TS3}$
Table 7
Representative term structure regression results

<table>
<thead>
<tr>
<th>Source of data</th>
<th>Results using spreads between yields</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Overnight &amp; three-month</td>
</tr>
<tr>
<td></td>
<td>tstat ($\beta_{TS1}$)</td>
</tr>
<tr>
<td>Actual data (based on Tables 1 and 2)</td>
<td>3.05*</td>
</tr>
<tr>
<td>Results from model simulations</td>
<td>Baseline model</td>
</tr>
<tr>
<td></td>
<td>No daily deviations (no FR1) [4, t = 0.0]</td>
</tr>
<tr>
<td></td>
<td>No near-term smoothing (no FR2) [$P^S(\tau) = P^d(\tau) = 0.03$]</td>
</tr>
<tr>
<td></td>
<td>No med.-term persistence (no FR3) [$P^S(\tau) = 0.03, P^d(\tau) = 0.005$]</td>
</tr>
</tbody>
</table>

Each column contains average t-statistics from a particular term structure regression. The first row gives averages of t-statistics reported in the literature. The remaining rows give averages of t-statistics obtained from model simulations.

*Significant at the 1 percent level.
Reconciling the Facts

- TS1 can be explained, partially, as deviations of the fed funds from target.
- However, notice that $\hat{\beta}_{TS1}$ is still significant.
- Short-term smoothing affects the predictability of TS2. With no smoothing, there is no predictability and $\hat{\beta}_{TS2}$ is insignificant.
- Inducing medium-term persistence improves medium-term predictability and can generate $\hat{\beta}_{TS3}$ that is significant.