Policy Rules for Inflation Targeting
by
Rudebusch and Svensson (1999) *NBER*

Overview

Present a small *empirical* model of the U.S. economy to examine the performance of inflation targeting rules.

Inflation-targeting is modeled using loss functions over policy goals – i.e. the usual quadratic loss function on inflation and output gap variability.

Classes of Rules:

1. *Instrument Rules*: expresses the m.p. instrument as an explicit function of available information.
2. *Targeting Rules*: based on deviations of a goal variable from target level.
Relation: a targeting rule is an implicit instrument rule in the context of a model of the economy. The solution of the model given the targeting rule delivers and explicit instrument rule.

The Model

Motivation:

1. Simplicity: choose a linear model and quadratic preferences.
2. Robustness to model variation.
3. Empirical fit to the data.
The Model

\[ \pi_{t+1} = \alpha_1 \pi_t + \alpha_2 \pi_{t-1} + \alpha_3 \pi_{t-2} + \alpha_4 \pi_{t-3} + \alpha_y y_t + \varepsilon_{t+1} \]
\[ y_{t+1} = \beta_1 y_t + \beta_2 y_{t-1} - \beta_r (\bar{i}_t - \bar{\pi}_t) + \eta_{t+1} \]

- \( \pi_t = 400(\ln p_t - \ln p_{t-1}) \), \( p \) is the GDP chain-weighted price index.
- \( \bar{\pi}_t = (1/4) \sum_{j=0}^{3} \pi_{t-j} \)
- \( i_t \) is the quarterly average of the fed funds rate
- \( \bar{i}_t = (1/4) \sum_{j=0}^{3} i_{t-j} \)
- \( y_t = 100(q_t - q_t^*) / q_t^*, q_t \) is real GDP, \( q_t^* \) is potential GDP
- All variables demeaned.
Remarks

• The model will use short-term rates as the instrument – no role for money aggregates.

• specified in terms of the output gap.

• the Phillips curve contains adaptive expectations terms but is otherwise consistent with the natural rate hypothesis.

• However, there are no forward-looking terms.

• The lags of inflation sum up to one, thus, the Phillips curve is vertical in the long-run.


\[ \pi_{t+1} = 0.70\pi_t - 0.10\pi_{t-1} + 0.28\pi_{t-2} + 0.12\pi_{t-3} + 0.14y_t + \varepsilon_{t+1}, \]

\[ (0.08)\quad (0.10)\quad (0.10)\quad (0.08)\quad (0.03)\]

\[ SE = 1.009,\quad DW = 1.99,\]

\[ y_{t+1} = 1.16y_t - 0.25y_{t-1} - 0.10(\bar{\pi}_t - \bar{\pi}_t) + \eta_{t+1}, \]

\[ (0.08)\quad (0.08)\quad (0.03)\]

\[ SE = 0.819,\quad DW = 2.05.\]
How good is this model?

• Compare with the MPS model (used by the FED): suppose the fed funds rate increased by 1% over two years and then it return to steady-state.

<table>
<thead>
<tr>
<th>Table 1. Model Responses to a Funds Rate Increase</th>
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<tbody>
<tr>
<td>(Annual average difference from baseline in percentage points)</td>
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<tr>
<td>Output Gap</td>
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<td>MPS</td>
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<td>Our Model</td>
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<td>Inflation</td>
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<td>MPS</td>
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<td>Our Model</td>
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• Compare the dynamic responses with those in a 3 variable VAR of $\pi$, $y$, and $i$. 
Figure 1
VAR and Structural Model Impulse Responses

Notes: The impulse responses of the structural model (amended with the VAR interest rate equation) are shown as the solid lines. The impulse responses of the VAR are shown as long-dashed lines and their 95 percent confidence intervals are shown as short-dashed lines.
Monetary Policy Rules

Given the expressions for the IS curve and the Phillips curve, an “inflation targeting” rule is specified with the loss function,

\[ E_t \sum_{\tau=0}^{\infty} \delta^\tau L_{t+\tau} \]

\[ L_t = \bar{\pi}_t^2 + \lambda y_t^2 + \nu (i_t - i_{t-1})^2 \]

where \( \bar{\pi}_t \) is in deviations from target.

Under certain conditions, it is useful to use the following simplification,

\[ E[L_t] = \text{var}[\bar{\pi}_t] + \lambda \text{var}[y_t] + \nu \text{var}[i_t - i_{t-1}] \]
The state-space representation of the model can be written as,

\[ X_{t+1} = AX_t + Bi_t + v_{t+1}. \]

with,

\[
X_t = \begin{bmatrix}
\pi_t \\
\pi_{t-1} \\
\pi_{t-2} \\
\pi_{t-3} \\
y_t \\
y_{t-1} \\
i_{t-1} \\
i_{t-2} \\
i_{t-3}
\end{bmatrix}, \quad A = \begin{bmatrix}
\pi_t \\
\pi_{t-1} \\
\pi_{t-2} \\
\pi_{t-3} \\
y_t \\
y_{t-1} \\
i_{t-1} \\
i_{t-2} \\
i_{t-3}
\end{bmatrix}, \quad \begin{bmatrix}
\sum_{j=1}^{4} \alpha_{ij} e_j + \alpha_y e_5 \\
e_1 \\
e_2 \\
e_3 \\
e_4 \\
e_5 \\
e_6 \\
e_7 \\
e_8 \\
e_9
\end{bmatrix}, \quad B = \begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0
\end{bmatrix}, \quad v_t = \begin{bmatrix}
e_t \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0
\end{bmatrix}
\]

where \( e_j \) (\( j = 0, 1, ..., 9 \)) denotes a 1x9 row vector, for \( j = 0 \) with all elements equal to zero, for \( j = 1, ..., 9 \) with element \( j \) equal to unity and all other elements equal to zero; and where \( e_{j:k} \) (\( j < k \)) denotes a 1x9 row vector with elements \( j, j+1, ..., k \) equal to \( \frac{1}{4} \) and all other elements equal to zero.
Similarly, define

\[ Y_t = C_X X_t + C_i i_t, \]

where

\[
Y_t = \begin{bmatrix}
\bar{\pi}_t \\
y_t \\
i_t - i_{t-1}
\end{bmatrix},
C_X = \begin{bmatrix}
e_{1:4} \\
e_5 \\
-e_7
\end{bmatrix},
C_i = \begin{bmatrix}
0 \\
0 \\
1
\end{bmatrix}
\]

with period loss function

\[ L_t = Y_t' K Y_t, \]

where \( K \) is a \( 3 \times 3 \) matrix of zeros with diagonal \((1, \lambda, \nu)\).
Given this set-up, Rudebusch and Svensson consider the following menu of rules:

1. **Linear Feedback Instrument Rules:** \( i_t = fX_t \)
2. **Optimal Instrument Rule:** the state-space form allows solving the model as a standard stochastic linear regulator problem.
3. **Inflation Forecasts:**
   a. *Constant-interest-rate forecasts*
   b. *Rule-consistent inflation forecasts*
4. **Simple instrument rules:**
   a. *Smoothing (S):* partial adjustment of \( i_t \)
   b. *Level (L):* no-smoothing.
   c. *Difference (D):* smoothing parameter is 1.
Figure 2
Policy Rule Frontiers

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Standard Deviation ($\bar{y}$) vs. Standard Deviation ($\bar{x}$)

- Optimal Rule
- Historical Data
- Estimated Taylor-type Rule
- Taylor Rule

- FIFT(12)
- FIFT(16)
- S($\pi_{t+1}$)
- S($\pi_{t+1}$)
- S($\pi_{t+1}$)
- S($\pi_{t+1}$)
- S($\pi_{t+1}$)
Results:

- Simple forward looking rules do well – they nearly match the optimal values.
- **Issues:** parameter and model uncertainty
- Simple instrument rules that respond only to inflation do quite poorly.
- inflation-forecast targeting rules do well if allowed to be sufficiently flexible.