1.(a) We have
\[
\begin{align*}
\hat{y}'A\hat{y} & = \hat{y}'A(y - \bar{u}) \text{ as } y = \hat{y} + \bar{u} \\
& = \hat{y}'Ay - \hat{y}'A\bar{u} \\
& = \hat{y}'Ay - \hat{y}'(I-\frac{1}{N})\bar{u} \\
& = \hat{y}'Ay - \hat{y}'\bar{u} \text{ as } I\bar{u} = 0 \\
& = \hat{y}'Ay \text{ as } \hat{y}'\bar{u} = \beta'X'\bar{u} = 0.
\end{align*}
\]

(b) Hence
\[
R^2 = \frac{\hat{y}'A\hat{y}}{y'y} = \frac{\hat{y}'Ay}{y'y} = \frac{\hat{y}'Ay \times \hat{y}'Ay}{(y'y)(\hat{y}'\hat{y})} = \frac{(\hat{y}'Ay)^2}{(y'y)(\hat{y}'\hat{y})}.
\]

(c) \(y'Ay = \sum_i(y_i - \bar{y})^2\).
Recall that \(A = A'A\) and \(Az = (I-\frac{1}{N})z = z - \bar{z}\).
Then \(\hat{y}'Ay = (A\hat{y})'Ay = (\hat{y} - \bar{y})(y - \bar{y}) = \sum_i(y_i - \bar{y})(y_i - \bar{y})\). So
\[
R^2 = \frac{(\hat{y}'Ay)^2}{(y'y)(\hat{y}'\hat{y})} = \frac{\left(\sum_i(y_i - \bar{y})(y_i - \bar{y})\right)^2}{(\sum_i(y_i - \bar{y})^2)(\sum_i(y_i - \bar{y})(y_i - \bar{y}))}
\]
Which is the squared correlation coefficient between \(\hat{y}\) and \(y\).

2.(a) Unbiasedness:
\[
\bar{\beta} = (Z'X)^{-1}Z'y \\
= (Z'X)^{-1}Z'[X\beta + u] \quad \text{given } y = X\beta + u \\
= (Z'X)^{-1}Z'\beta + (Z'X)^{-1}Z'u \\
= \beta + (Z'X)^{-1}Z'u
\]
\[
E[\bar{\beta}] = E[\beta + (Z'X)^{-1}Z'u] \\
= E[\beta] + E[(Z'X)^{-1}Z'u] \\
= \beta + (Z'X)^{-1}Z'E[u] \quad \text{given } Z \text{ and } X \text{ nonstochastic} \\
= \beta + (Z'X)^{-1}Z' \times 0 \quad \text{given } E[u] = 0 \\
= \beta
\]

(b) Variance matrix
\[
V[\bar{\beta}] = E[(\bar{\beta} - \beta)(\bar{\beta} - \beta)'] \\
= E\{((Z'X)^{-1}Z'u) \times ((Z'X)^{-1}Z'u)\}' \quad \text{given } y = X\beta + u \\
= E[(Z'X)^{-1}Z'\times uZ(X'X)^{-1}] \quad \text{transposing} \\
= (Z'X)^{-1}Z'E[uu']Z(X'X)^{-1} \quad \text{given } Z \text{ and } X \text{ nonstochastic} \\
= (Z'X)^{-1}Z'\sigma^2IZ(X'X)^{-1} \quad \text{given } V[u] = E[uu'] = \sigma^2I \\
= \sigma^2(Z'X)^{-1}Z'Z(X'X)^{-1}
\]

(c) If \(u\) is multivariate normally distributed then \(\bar{\beta} = \beta + (Z'X)^{-1}Z'u\), which is a linear transformation of \(u\), will be multivariate normally distributed.
(d) We have

\[ B^{-1} - A^{-1} = \left( \sigma^2 (X'X)^{-1} \right)^{-1} - \left( \sigma^2 (Z'X)^{-1} Z'(X'Z)^{-1} \right)^{-1} \]

\[ = \sigma^{-2} (X'X) - \sigma^{-2} X'Z'(Z'Z)^{-1}Z'X \]

\[ = \sigma^{-2} X' \{ I - Z(Z'Z)^{-1}Z' \} X \]

\[ = \sigma^{-2} X'MX \text{ where } M = I - Z(Z'Z)^{-1}Z' \]

\[ = \sigma^{-2} X'M'MX \text{ as } M'M = M \]

\[ = \sigma^{-2} (MX)'(MX) \text{ which is a positive semi-definite matrix.} \]

(e) It follows that \( B - A \) is a negative semidefinite. 

i.e. \( V[\tilde{\beta}_{OLS}] - V[\tilde{\beta}] \) is negative semidefinite. The OLS estimator is at least as efficient as \( \tilde{\beta} \).

3.(a) We have

\[ \tilde{\beta} = (X'AX)^{-1}X'Ay \]

\[ = (X'AX)^{-1}X'A[X\beta + u] \]

\[ = \beta + (X'AX)^{-1}X'Au \]

\[ \Rightarrow \tilde{\beta} - \beta = (X'AX)^{-1}X'Au. \]

(b) And in general

\[ \tilde{u} = y - X\tilde{\beta} \]

\[ = X\beta + u - X\tilde{\beta} \]

\[ = u - X(\tilde{\beta} - \beta) \]

\[ = u - X(X'AX)^{-1}X'Au \]

\[ = [I - X(X'AX)^{-1}X'A]u. \]

(c) And

\[ E[\tilde{u}'\tilde{u}] = E[(Mu)'(Mu)] \]

\[ = E[u'M'Mu] \]

\[ = E[u'Pu] \]

\[ = E[\sum_i \sum_j P_{ij}E[u_iu_j] \]

\[ = E[\sum_i P_{ii}\sigma^2] \text{ using } u \sim [0, \sigma^2 I] \]

\[ = \sigma^2 \text{trace}[P]. \]

(d) And

\[ \text{trace}[P] = \text{trace}[M'M] \]

\[ = \text{trace}[(I - AX(X'AX)^{-1}X')(I - X(X'AX)^{-1}X'A)] \text{ using } A = A' \]

\[ = \text{tr}[I - AX(X'AX)^{-1}X' - X(X'AX)^{-1}X'A + A'X(X'AX)^{-1}X'X(X'AX)^{-1}X'A] \]

\[ = \text{tr}[I - \text{tr}[X'AX(X'AX)^{-1}]] - \text{tr}[(X'AX)^{-1}X'AX] + \text{tr}[X'AA'X(X'AX)^{-1}X'X(X'AX)^{-1}] \]

\[ = N - k - k + \text{tr}[X'X(X'AX)^{-1}] \text{ using } AA' = A \]

\[ \neq N - k \text{ necessarily.} \]
4.(a) \( H_0 : \beta_{\text{exper}} = 0 \) against \( H_a : \beta_{\text{exper}} \neq 0 \).

Wald test statistic: 8.5431703 with p-value: .00403303

Reject \( H_0 \) as \( p < 0.05 \). Conclude exper is statistically significant at 5%.

Note that same p-value as for the t-statistic in the regular regress output.

(b) \( H_0 : \beta_2 = 0, \ldots, \beta_k = 0 \) against \( H_a : \) at least one \( \beta_j \neq 0 \).

Wald test statistic: 10.816136 with p-value: 6.624e-10

Reject \( H_0 \) as \( p < 0.05 \). Conclude regressors are jointly statistically significant at 5%.

Note that same F statistic and p-value as for the F-statistic in the regular regress output.

(c) \( H_0 : \beta_{\text{exper}} = 0, \beta_{\text{expersq}} = 0 \) against \( H_a : \) at least one of these \( \neq 0 \).

Wald test statistic: 11.766523 with p-value: .00001859

Reject \( H_0 \) as \( p < 0.05 \). Conclude the two regressors are jointly statistically significant at 5%.

(d) \( H_0 : \beta_{\text{union}} = \beta_{\text{educ}} \) or \( H_0 : \beta_{\text{union}} - \beta_{\text{educ}} = 0 \) against \( H_a : \beta_{\text{union}} - \beta_{\text{educ}} \neq 0 \)

Wald test statistic: .58357782 with p-value: .44617195

Do not reject \( H_0 \) as \( p > 0.05 \). Conclude the two regressors have different impact.

(e) test exper: F( 1, 143) = 8.54 with Prob > F = 0.0040
test jirate exper expersq educ union nonwhite: F( 6, 143) = 10.82 Prob > F = 0.0000
test exper expersq F( 2, 143) = 11.77 Prob > F = 0.0000
test educ = union F( 1, 143) = 0.58 Prob > F = 0.4462

5.(a) matrix R=(0,0,1,0,0,0,0,0,0,0,0,1,0,0,0)
matrix r=(0,0,0,0)
mkmat cont jirate exper expersq educ union nonwhite, matrix(X)
mkmat lnwage
matrix betahat=syminv(X'*X)*X'*lnwage
matrix b=betahat-syminv(X'*X)*R'*syminv(R*syminv(X'*X)*R')*(R*betahat-r)
matrix list b
    c1
    cont 1.8075105
    jirate -.02602483
    exper 5.204e-17
    expersq -8.132e-19
    educ .06256398
    union .19465501
    nonwhite -.28553339

(b) regress lnwage jirate educ union nonwhite
The results are the same.

(c) F=((26.26-22.55)/2)/(22.55/143)=11.77, the same as in 4(c).

6.(a) regress wage jirate exper expersq educ union nonwhite
lincom 3*jirate+10*exper+100*expersq+12*educ+1*union+0*nonwhite+_cons
wage=13.83649,

(b) standard error=1.955859

(c) [13.8365-1.96*1.9559, 13.84+1.96*1.9559].

(d) \[(1.9559^2+57.705547)^{1/2}=7.84\]