1. Nonlinear Regression - marginal effects
Suppose we obtain model estimates that yield predicted conditional mean
\[ \hat{E}[y|x] = \exp(1 + 0.01x) / [1 + \exp(1 + 0.01x)]. \]
Suppose the sample is of size 100 and \( x \) takes integer values 1, 2, ..., 100. Obtain the following estimates of the marginal effect
\[ \frac{\partial \hat{E}[y|x]}{\partial x} \]
(a) The average marginal effect over ALL observations.
(b) The marginal effect of the average observation.
(c) The marginal effect when \( x = 90 \).
(d) The marginal effect of a one-unit change when \( x = 90 \) to \( x = 91 \).

2. Maximum Likelihood Estimation
Consider the special one-parameter case of the gamma distribution
\[ f(y) = \frac{y}{\lambda^2} \exp\left(\frac{-y}{\lambda}\right), y > 0, \lambda > 0. \]
For this distribution it can be shown that
\[ E[y] = 2\lambda; V[y] = 2\lambda^2. \]
Here we introduce regressors and suppose that in the true model the parameter \( \lambda \) depends on regressors according to
\[ \lambda_i = \frac{\exp(x_i\beta)}{2}. \]
Thus
\[ E[y_i|x_i] = \exp(x_i\beta) \text{ and } V[y_i|x_i] = \frac{[\exp(x_i\beta)]^2}{2}. \]
Assume that the data \( \{y_i, x_i\}_{i=1}^n \) are a random sample.
(a) Show that the log-likelihood function (scaled by \( n^{-1} \)) for this gamma model is
\[ Q_n(\beta) = \frac{1}{n} \sum_{i=1}^{n} \{ \ln y_i - 2x_i\beta + 2\ln 2 - 2y_i \exp(-x_i\beta) \} \]
(b) Show that
\[ \frac{\partial Q_n(\beta)}{\partial \beta} = \frac{1}{n} \sum_{i=1}^{n} 2 \left( \frac{(y_i - \exp(x_i\beta))}{\exp(x_i\beta)} \right) x_i \]
(c) Prove that \( \hat{\beta} \) that is the local maximum for \( Q_n(\beta) \) is consistent for \( \beta_0 \), the true value of \( \beta \).
State any assumptions made.
(d) What essential condition indicated by the first order conditions in (b) needs to be satisfied for \( \hat{\beta} \) to be consistent?

(e) Obtain 

\[
\frac{\partial^2 Q_n(\beta)}{\partial \beta \partial \beta} \bigg|_{\beta = \beta_0}
\]

and obtain its limit in probability.

(f) Hence obtain the distribution of \( \sqrt{n}(\hat{\beta} - \beta_0) \) as \( n \to \infty \).

3. The Logit Model
Consider the logit model with

\[
P[y = 1|x_1, x_2] = \Lambda(\beta_0 + \beta_1 x_1 + \beta_2 x_2); \quad \Lambda(z) = \frac{e^z}{1 + e^z}.
\]

(a) Write down the likelihood scores and information matrix in expanded form.

(b) Use these to derive Wald and LM score tests of \( H_0 : \beta_2 = 0 \).

(c) Explain how you would computationally implement these tests.

(d) In what sense is the logit model intrinsically heteroskedastic?

4. Instrumental Variable Regression
The following question is about estimating a labor supply/demand curve (you will have to tell me which) and uses data in the file \( \text{ww2-state-mob-rates.xlsx} \). Nearly 16 million men were mobilized to serve in the Armed Forces during World War II, and around 73% of them were sent overseas. This shrinking in the number of male workers drew many women into the civilian labor force. Farm states saw less men mobilized than other states, such as Massachussets and New York, because producing food was obviously a priority. Other factors along the same lines explain variation in mobilization rates across states. The data in the excel file contains observations for the variable \( \text{mob-rate} \) which refers to the proportion of registered men ages 18-44 who served in the military between 1940 and 1945. The variable \( \% \text{ change in female employment} \) and the variable \( \% \text{ change in female wage} \) are self explanatory and refer to changes between 1939-1949. You can ignore the other variables. The source of the data is from a study by Acemoglu, Autor and Lyle (2004) “Women, War and Wages: The Effects of Female Labor Supply on the Wage Structure at Midcentury,” \textit{Journal of Political Economy}, 112: 497-551. \textbf{Note:} be careful in how you read the data into STATA since there is a missing observation, the data is not in .csv format, the labels for the variables may not be read-in correctly by STATA, and some of the variables are not used.

(a) Regress \( \% \text{ change in female employment} \) and \( \% \text{ change in female wage} \) on \( \text{mob-rate} \) (i.e., in two separate regressions). Comment on the economics behind the results you just obtained.

(b) Regress \( \% \text{ change in female employment} \) on to \( \% \text{ change in female wage} \) using OLS. Explain whether or not the coefficient estimate makes economic sense to you and discuss whether you can or cannot interpret this coefficient as the labor demand elasticity for female employment.

(c) Repeat the regression in part (b) using \( \text{mob-rate} \) as an instrument for \( \% \text{ change in female wage} \). Is this a weak instrument based on your regression in part (a)? Is this a valid instrument based on economic reasoning (you do not have overidentifying restrictions here to do the usual test)? Compare the coefficient estimate of \( \% \text{ change in female wage} \) with that obtained in part (b). What do you attribute the differences to? What is the economic interpretation of the regression?

(d) Show that the ratio of the slope coefficients in the regressions of part (a) is the same as the coefficient in the IV regression in part (c). Explain why this is so.
5. Limited Dependent Variables

The data for this exercise is in the file participation.data which is taken from Gerfin (1996) and contains observations for 872 Swiss women who may or may not participate in the labor force. The variables are arranged in columns as follows:

- column 1: observation
- column 2: labor force participation (1 for in, 0 for out)
- column 3: log of nonlabor income
- column 4: age/10
- column 5: education in years
- column 6: number of children under 7 years of age
- column 7: number of children over 7 years of age
- column 8: citizenship dummy (1 for Swiss, 0 otherwise).

In addition, generate a new regressor that consists in squaring a woman’s age. **Note:** the data is reported in a text file where columns are separated by spaces. Be sure that the data loads correctly into STATA.

**a)** Estimate a linear probability model, a probit and a logit to explain a woman’s decision to participate in the labor force.

**b)** What is the predicted probability that a woman will participate in the labor force at age 30 if she has 5 years of education, no children and is a citizen using each of the models in part (a)?

**c)** What is the marginal effect on the probability of participation in the labor force of an extra year of education at the average value of the regressors?

**d)** How would your answer in part (b) change if the individual had 15 years of education instead? How is your answer related to your answer in (c)?