Macroeconomic Modeling for Monetary Policy Evaluation
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This paper presents the basic model used by most central banks in discussing monetary policy.

It represents the modern version of the IS-LM model.

We still have 3 equations, but these are derived in an optimizing framework within an economy characterized by uncertainty

Important differences between IS-LM and modern framework

Behavioral equations are derived from dynamic optimization under uncertainty. This has three important implications

1. The current **and expected path** of short term real interest rates effect economic activity.
2. The effectiveness of monetary policy depends critically on the credibility of the monetary authority (central bank).
3. Parameters of the model reflect parameters describing tastes and technology. Hence, stable with respect to a change in monetary policy (Lucas critique).

Monetary Policy is characterized by an interest rate rule (the Taylor rule) rather than the money supply.

Gali-Gertler, p A1, description of consumption

A representative household makes consumption, saving, and labor supply decisions in this environment. Within this frictionless setting, the permanent income hypothesis holds perfectly. An implication is that the household strictly obeys a conventional Euler equation that relates the marginal cost of saving (the foregone marginal utility of consumption) to the expected marginal benefit (the expected product of the ex post real interest rate and the discounted marginal utility of consumption in the next period). Log-linearizing this equation yields a familiar positive relation between expected consumption growth and the ex ante real interest rate: Everything else equal, an expected rise in real rates makes the return to saving more attractive, inducing households to reduce current consumption relative to expected future consumption.

The consumption Euler equation

Consider the maximization problem of choosing assets and consumption every period:

$$\max E\left[\sum_{t=1}^{\infty}\beta^{t-1}U\left(c_t\right)\right]$$

$$subject\ to\ y_t + A_{t-1}rr_{t-1} = c_t + A_t$$

$\beta =$ subjective discount factor

$y_t =$ income

$A_{t-1} =$ assets purchased in previous period

$rr_t =$ real return on assets (known at time t)

The optimal choice of consumption and assets will be characterized by the following necessary condition – this is the Euler Equation:

$$U'(c_t) = \beta E_t \left[U'(c_{t+1})\right] rr_t$$

What does this say? Suppose you want to buy 1 more asset. This means you reduce your consumption today by 1 – the cost is the current marginal utility. (The LHS).

The gain is the extra return next period, rr(t). How much you value this in utils is given by the product of MU and the return. This is then discounted back to compare values.

At the optimum: MC = MB

As an aside – this is the starting point for modern finance theory.

Now – we know the Euler equation. What does log-linearize mean?

We want to use the Euler equation but this is difficult since it is inherently non-linear.

Convert to a linear expression by taking a first-order Taylor series approximation around full employment levels of consumption and returns (described below). This is the linearize part.

The log part means we will express all variables as percentage deviations from full employment.

Recall from your calculus that a function can be approximated by a Taylor series approximation:

$$f\left(x\right) = f\left(x_0\right) + f'\left(x_0\right)\left(x - x_0\right) + f''\left(x_0\right)\frac{\left(x-x_0\right)^2}{2} + ..$$

We will use a first-order Taylor series approximation – this turns the equation into a linear equation.

The method: take the derivative of the expression and evaluate the derivative at the full employment levels. Then multiply this by the difference between the variable at time t and the full employment level.

We will repeatedly use this term:

$$f'(x_0)(x - x_0)$$

Assume that the utility function is: $U(c_t) = \frac{c_t^{1-\gamma}}{1-\gamma}$

Then the marginal utility is: $c_t^{-\gamma}$

Take the first-order Taylor series approximation around c_f

$$\text{approximation } = c_t^{-\gamma} \approx c_f^{-\gamma} - \gamma c_f^{-\gamma-1}(c_t - c_f)$$

Back to the Euler Equation

$$U'(c_t) = \beta E_t \left[U'(c_{t+1})\right] rr_t$$

$$c_t^{-\gamma} = \beta E_t \left[c_{t+1}^{-\gamma}\right] rr_t$$

$$rr_f = \text{full employment interest rate}$$

$$c_f^{-\gamma} = \beta \left[c_f^{-\gamma}\right] rr_f \Rightarrow \beta rr_f = 1$$

Linearize the Euler equation

$$c_t^{-\gamma} = \beta E_t \left[c_{t+1}^{-\gamma}\right] rr_t$$

$$c_f^{-\gamma} - \gamma c_f^{-\gamma-1}(c_t - c_f) = \beta \left[c_f^{-\gamma}\right] rr_f +$$
$$\beta rr_f E_t \left[-\gamma c_f^{-\gamma-1}(c_{t+1} - c_f)\right] + \beta c_f^{-\gamma}(rr_t - rr_f)$$

Note this can be written as:

$$-\gamma c_f^{-\gamma} \frac{(c_t - c_f)}{c_f} = \beta rr_f E_t \left[-\gamma c_f^{-\gamma} \frac{(c_{t+1} - c_f)}{c_f}\right] + \beta rr_f c_f^{-\gamma} \frac{(rr_t - rr_f)}{rr_f}$$

Define variables as percentage deviation from full employment:

$$\frac{(c_t - c_f)}{c_f} = \tilde{c}_t = \text{consumption as \% deviation from full employment}$$

Cancel common terms and use result from full employment interest rate.

Log-linearized Euler equation

$$-\gamma \tilde{c}_t = -\gamma E_t\left(\tilde{c}_{t+1}\right) + \tilde{rr}_t$$

Or, dividing by gamma, we have:

$$\tilde{c}_t = E_t\left(\tilde{c}_{t+1}\right) - \sigma \tilde{rr}_t$$

$$\frac{1}{\gamma} = \sigma = \text{intertemporal elasticity of substitution}$$

Finally, use the Fisher relationship: The real interest rate is equal to the nominal interest rate minus the expected inflation rate:

$$rr_t = r_t - E_t\left(\pi_{t+1}\right)$$

$$c_t = -\sigma\left(r_t - E_t\left(\pi_{t+1}\right)\right) + E_t\left(c_{t+1}\right)$$

See eq. (AD2) on p. A2 in the article

Consumption is determined by the path of interest rates!

Recursively update the Euler equation

$$c_t = -\sigma\left(r_t - E_t\left(\pi_{t+1}\right)\right) + E_t\left(c_{t+1}\right)$$

$$c_{t+1} = -\sigma\left(r_{t+1} - E_{t+1}\left(\pi_{t+2}\right)\right) + E_{t+1}\left(c_{t+2}\right)$$

$$c_t = -\sigma\left(r_t - E_t\left(\pi_{t+1}\right)\right) + E_t\left[-\sigma\left(r_{t+1} - E_{t+1}\left(\pi_{t+2}\right)\right)\right] + E_t\left[c_{t+2}\right]$$

Consumption depends on current and expected real interest rates

$$c_t = -\sigma \textstyle\sum_{i=0}^{\infty} E_t\left[r_{t+i} - \pi_{t+1+i}\right]$$

Expressing in deviation from steady-state form:

$$\tilde{c}_t = -\nu_c r\tilde{r}_t^l$$

where $r\tilde{r}_t^l = \sum_{t=0}^{\infty} E_t\left(rr_{t+i}\right)$

Almost have AD described!

Recall the identity: output = consumption plus investment

$$y_t = c_t + i_t$$

Or, in deviation from full employment form

$$\tilde{y}_t = \theta_c \tilde{c}_t + \nu_i \tilde{\imath}_t$$

Using the result we just obtained

$$\tilde{y}_t = -\nu_c r \tilde{r}_t^l + \nu_i \tilde{\imath}_t$$

AD depends on the path of interest rates….now need to describe investment

Investment Demand

Household's own capital and labor and supply these to firms:

$$\max_{(c_t, h_t, i_t)} E_t \sum_{t=0}^{\infty} \beta^t \left[U(c_t) + V(h_t) \right]$$

Subject to income constraint:

$$R_t k_t + W_t h_t = P_t c_t + P_t i_t$$

And law of motion for capital:

$$k_{t+1} = (1-\delta) k_t + \epsilon_{i,t} \left[1 - \frac{\varphi}{2} \left(\frac{i_t}{k_t} - \delta \right)^2 \frac{k_t}{i_t} \right] i_t$$

And $\epsilon_{i,t}$ is the exogenous investment adjustment cost shock.

φ is the adjustment cost parameter

First-order condition for optimal investment

$$1 = q_t \epsilon_{i,t} \left[1 - \varphi \left(\frac{i_t}{k_t} - \delta \right) \right]$$

q_t is the price of capital in terms of consumption

MC = MB interpretation

LHS is the cost of consumption while the RHS is the additional capital that will be available. Note that if the shock is always equal to 1 and adjustment costs are zero, then capital will have the same price as consumption (equal to one).

Log linearize the first order condition

I will spare you the details:

$$1 = q_t \epsilon_{i,t} \left[1 - \varphi\left(\frac{i_t}{k_t} - \delta\right)\right]$$

This becomes (see eq. AD3 on p. A2):

$$0 = \widehat{q_t} + \widehat{\epsilon_{i,t}} - \varphi \widehat{\left(\frac{i_t}{k_t}\right)}$$

$$\left(\widehat{i_t} - \widehat{k_t}\right) = \eta \widehat{q_t} + \eta \widehat{\epsilon_{i,t}}, \text{ where } \eta = 1/\left(\delta\varphi\right)$$

If capital does not change much over the business cycle, then This eq. describes investment. Use that in the definition of AD

THE AD Equation...modern IS curve

Recall we had

$$\tilde{y}_t = -\nu_c r \tilde{r}_t^l + \nu_i \tilde{\imath}_t$$

But we just showed that:

$$\tilde{\imath}_t = \eta \tilde{q}_t$$

The Modern version of IS curve

$$\tilde{y}_t = -\nu_c r \tilde{r}_t^l + \nu_i \eta \tilde{q}_t$$

Modern IS curve

$$\tilde{y}_t = -\nu_c r \tilde{r}_t^l + \nu_i \eta \tilde{q}_t$$

Key points

1. Current and future interest rates affect aggregate demand. The yield curve is important!

2. The price of capital, $\tilde{q}_t$, also affects AD. This is affected by shocks to investment and future productivity of capital (we have not shown this).

3. The parameters in the equation are derived from optimization and reflect preferences and technology.