1 Homework #1 - Answers.

1. **Direct Finance**: where the two parties, buyer and seller, meet directly. No change in the nature of the asset (i.e. maturity, denomination, etc.) **Indirect finance**: Buyers and sellers of assets trade with an intermediary which typically changes the nature of the initial asset.

2. **Moral Hazard**: the provision of insurance changes the probability of loss. That is, if an agent purchases insurance, his or her actions will increase the likelihood of a loss. The problem is that behavior cannot be directly observed so a contract cannot be written on behavior, only outcomes. **Adverse Selection**: In markets with goods that have different characteristics which are unobservable (e.g. quality of a used car), the price will reflect the average quality of the goods being sold. But high quality sellers may, therefore, decide to not sell their goods. The result is only low quality goods are sold or, in the case of lending, only high risk borrowers take loans.

3. The implied inflation rate is 20% over the 4 year period so a 5% annual rate (simple interest is fine for this answer). Hence the path of one-period current and expected nominal interest rates is (9%, 10%, 7%, 6%). Using the arithmetic approximation, the long rate is just the average of current and expected rates. Consequently, the current yield on the 2-year bond is 9.5%, the 3-year yield is 8.67%, while the 4-year yield is 8%.

4. The bond pricing formula is
   \[ P_b = F \left( \frac{r}{i} \left( 1 - \left( \frac{1}{1+i} \right)^N \right) \right) + \left( \frac{1}{1+i} \right)^N \]
   where \( P_b \) is the price of the bond, \( N \) is maturity, \( F \) is face (or par) value, \( i \) is the yield to maturity, and \( r \) is the coupon rate. \((N, r, i)\) must be adjusted for the frequency of the coupon payments.
   For this problem we have quarterly payments so, given the numbers provided, \( N = 20 \), \( i = 0.025 \), \( F = 1000 \), \( P_b = 900 \). Solving yields \( r = 0.0186 \) but this is on a quarterly basis so the actual coupon rate is 0.0744. Note that since the price is below the face value, we have \( r < i \).

5. First you use the bond pricing formula (adjusted for monthly payments) to get \( P_b = 1688.26 \). Then, using the duration formula and again making the adjustments since these are monthly payments, we get \( D = 6.38 \). So in annual terms \( D = 6.38/12 = 0.532 \). The duration is roughly one-half of a year. The change in the price of the bond is
   \[ \Delta P_b = -\frac{D}{1+i} (\Delta i) P_b = -16.03 \]
   If you recalculate the price of the bond, the actual change is -17.64 so as an approximation the duration formula is not too bad.

6. First calculate the new price of the bond with 39 more 6 month payment periods to go and a yield to maturity of 0.10. This yields \( P_b = 1170.17 \). Then the rate of return (expressed as an annual rate) is:
   \[ rate \ of \ return = 2 \left( \frac{60 + 1170.17}{1300} - 1 \right) = -0.107 \]
   Joe lost almost 11% on this investment.
7. Use the formula
\[
NIM = \left( \frac{ROE}{1-t} \right) \left( \frac{E}{IEA} \right) + \left( \frac{LLP - OI + OE + Dep}{IEA} \right)
\]
For this problem we have: \( ROE = 0.15, IEA = 100, E = 12, t = 0.30, Dep = 3, LLP = OI = OE = 0 \). This implies \( NIM = 0.0557 \).

8. We know that \( \text{Assets} / \text{Equity} = 15 \). But it is also true that \( \text{Assets} - \text{Debt} = \text{Equity} \). This implies \( \text{Debt} / \text{Assets} = 0.933 \).

9. Consider the evolution of prinicipal (i.e. the value of the bond) for the first three periods:
\[
P_1 = P_0 (1 + i) - C \\
P_2 = P_1 (1 + i) - C \\
P_3 = P_2 (1 + i) - C
\]
These can be combined to yield:
\[
P_3 = P_0 (1 + i)^3 - C \left[ 1 + (1 + i) + (1 + i)^2 \right]
\]
Or, in general, for any \( T \) periods ahead the value is (make sure you understand the summation term...set \( T = 3 \) and break out the individual terms):
\[
P_T = P_0 (1 + i)^T - C \left( \sum_{j=1}^{T} (1 + i)^{T-j} \right)
\]
Factor out the term \( (1 + i)^T \) from the summation term to yield (again, make sure the indexing term in the summation makes sense):
\[
P_T = P_0 (1 + i)^T - (1 + i)^T \sum_{j=1}^{T} \left( \frac{1}{1+i} \right)^j
\]
But in class we demonstrated (in solving for the bond pricing formula) that the summation is equal to:
\[
\sum_{j=1}^{T} \left( \frac{1}{1+i} \right)^j = \frac{1}{i} \left[ 1 - \left( \frac{1}{1+i} \right)^T \right]
\]
Therefore the value of the principal in period \( T \) is:
\[
P_T = P_0 (1 + i)^T - C \frac{1}{i} \left( 1 + i \right)^T \left[ 1 - \left( \frac{1}{1+i} \right)^T \right]
\]
Finally, since the loan must be paid off when \( T = N \), that is \( P_T = 0 \), we have
\[
P_0 = C \frac{1}{i} \left[ 1 - \left( \frac{1}{1+i} \right)^T \right]
\]
This is the same formula is in question #4 but when \( C = rF \) and there is no final payment of \( F \).