1. See attached charts. Note the recession and financial crisis have taken a toll on ROE and ROA. Also note that, prior to the current recession, the ROE on large banks was greater than that of small banks despite having similar ROA’s. What does this imply. Also note that NIM for large banks is typically smaller than that for small banks. Large banks do not have access to the volume of low cost deposits that small banks do.

2. Solve for $x$: $x(7.3) + (1 - x)(2.2) = 5$. This yields $x = 0.54902$ implying $549,020$ is placed in the bonds with 7.3 duration and the remaining is invested in the other bond.

3. Describe (with explanation) the position one would take in the financial futures market in order to hedge interest rate risk implied by the following:

   (a) A bank has a negative duration GAP.
   
   ANSWER: With a negative gap, net worth will be hurt if interest rates fall (why?). So take a position that will make money in the futures market if interest rates fall or bond prices rise. Buy futures - Go long.

   (b) A S&L has a positive funding GAP.
   
   ANSWER: With a positive funding gap, net income will be hurt if interest rates fall (why?). Go Long

   (c) A mortgage firm plans to lend $3 million in 3 months.

4. Suppose the current spot price of wheat is $10/bushel while the futures price for wheat with delivery date of 1 year from now is $15/bushel. If there are no storage costs and the current one-year interest rate is 5%, construct an arbitrage that would generate profits.

   ANSWER: Today: borrow $10 and buy a bushel of wheat in the spot market. At the same time sell a futures contract for $15/bushel with delivery a year from now. In one year: Deliver wheat on futures contract and collect $15. Pay back the loan with interest ($10.50). Pocket the $4.50 as profit.

5. Suppose the current 1 year T-Bill rate is 4% while the 6 month T-bill rate is 3%. If the current futures price of a 6-month T-bill with delivery 6 months from now is $98.53 (per $100 of contract), demonstrate that this can not be an equilibrium (i.e. one can construct an arbitrage with positive returns).

   ANSWER: The price of the 1-year T-Bill (assume throughout that the face value is $100) = \frac{100}{1.04} = $96.15. At time t: Borrow $96.15 and buy a 1 year T-Bill. At the same time, sell a 6-month T-Bill futures contract with delivery in 6 months. In six months. Deliver the T-Bill on the 6 month futures contract (after 6 months, the 1-year T-bill is the same as a 6 month T-bill) and receive $98.53. Use this to pay back the loan which is $96.15 \times (1.03)^0.5 = $97.58. You pocket $0.95 with no risk!!.

6. Describe the strengths and weaknesses of funding GAP and duration GAP analyses.

   ANSWER: Funding GAP is easy to compute and measures the interest rate risk exposure of net interest income. The problems are that the definition of rate sensitive/fixed rate is arbitrary and that it ignores interest rate risk of net worth. DGAP measures the interest rate risk of equity (net worth) but is difficult to compute because one must have estimates of the duration of all assets and liabilities. It also needs to be recalculated over time since the duration of different maturity assets and liabilities change at different rates over time (duration drift).

7. In general, what are the advantages of a futures contract over a forward contract?

   ANSWER: The main advantage is that the futures contract is traded on an exchange. This means (1) that you can always buy and sell a contract (there is another side to the market) and (2) it reduces risk of default (i.e. counterparty risk).

8. Suppose Corp. A can borrow long term at a fixed rate of 9% or at a floating rate of 60 bp over LIBOR. Corp B can borrow long term at a fixed rate of 11% and a floating rate of 75bp over LIBOR. If Corp. A desires a floating rate and Corp. B wants a fixed rate, design an interest rate swap that reduces the borrowing costs of both firms.

   ANSWER: Corp. A has a comparative advantage in fixed rate. So it issues fixed rate at 9% while Corp B issues floating rate at $L + 0.75$. Assume that the swap deal has Corp A paying a floating rate of $LIBOR$ to Corp B. We need to determine a fixed swap rate (paid by Corp B) that is agreeable to both parties. The all-in-costs to Corp A are: $AIC = 9\% + L - SR$. This must be less than the floating rate they would get in the market place: $AIC < L + 0.60 \Rightarrow 8.40 < SR$. The all-in-costs to Corp B are: $AIC = L + 0.75 + SR - L$. These must be less than the fixed rate they could get in the market place: $AIC < 11 \Rightarrow SR < 10.25$. Hence any swap rate in the range: $8.40 < SR < 10.25$ would be acceptable to both parties.
9. Suppose that you are offered a 12% fixed rate payment over the next 15 years. The duration of the swap is -6.4. You are managing a firm with $3.7 million of assets and a duration gap of 1.7 years. How much notional principal would you want the swap to have?

ANSWER: You need the absolute change in the value of the swap to equal the absolute change in the value of equity. This requires:

\[ V_{\text{swap}}D_{\text{swap}} = -V_A DG\text{AP} \]

Solving for \( V_{\text{swap}} \) using the numbers provided in the question yields: \( V_{\text{swap}} = \frac{-3.7(1.7)}{-6.4} = 0.983 \) or $983,000 (with rounding).

10. Your portfolio consists of three assets that have stochastic returns. It is assumed that the returns for each asset is distributed normally with a mean of \( \mu \) and a standard deviation of \( \sigma \). The returns are assumed to be uncorrelated. If you have invested equally in the three assets, calculate the expected value (i.e mean) and standard deviation of the return on your portfolio. What is the implication for the expected value and standard deviation of the returns on your portfolio if you invest in \( N \) of these assets and \( N \) becomes very large?

ANSWER: Recall from Stats 13 the following relation. Suppose that \( x \) and \( y \) are random variables with means of \( \mu_x \) and \( \mu_y \) respectively and standard deviations of \( \sigma_x \) and \( \sigma_y \) respectively and, finally, have covariance of \( \sigma_{xy} \). Define the new random variable \( z = ax + by \) where \( a \) and \( b \) are constants. Then \( \mu_z = a\mu_x + b\mu_y \) and \( \sigma_z^2 = a^2\sigma_x^2 + b^2\sigma_y^2 + 2ab\sigma_{xy} \). (You should be able to show this.) This generalizes to a random variable defined as a linear function of \( n \) random variables; if \( w = \alpha_1x_1 + \alpha_2x_2 + \ldots + \alpha_nx_n \) then \( \mu_w = \sum_{i=1}^{n} \alpha_i\mu_i \) and \( \sigma_w^2 = \sum_{i=1}^{n} \alpha_i^2\sigma_i^2 + 2\sum_{i=1}^{n}\sum_{j=1}^{n} \alpha_i\alpha_j\sigma_{ij} \). (Show this for the three asset case as in this problem) For the problem, the returns to the portfolio are given by \( \rho_P = \frac{1}{3} r_1 + \frac{1}{3} r_2 + \frac{1}{3} r_3 \) since wealth is distributed equally among the three assets. Given that the three assets have the same return and standard deviation and no covariance, the above formula implies \( \mu_P = \mu \) (the portfolio return has the same mean as the underlying assets) and \( \sigma_P^2 = \frac{\sigma^2}{3} \) or standard deviation of the portfolio is \( \sigma_P = \frac{\sigma}{\sqrt{3}} \). The implication is that if the number of uncorrelated assets in your portfolio gets larger, then the standard deviation of the portfolio returns goes to zero.

The following are from Chapter 25 in your text:

14. A hedger takes a short position in five T-bill futures contracts at the price of 98.5/32. Each contract is for $100,000 principal. When the position is unraveled, the price is 95+12/32. What is the gain/loss on this transaction?

ANSWER: Gain per contract \((98.5/32 - 95+12/32) \times 1000 = 2781.25\). Total gain is 2781.25 \times 5 = 13,906.25 (note that the gain is multiplied by 1000 since the contract price is specified per $100: $100 \times 1000 = $100,000)

19. A bank issues a $3 million commercial mortgage with a nominal APR of 8%. The loan is fully amortized over 10 years requiring monthly payments. The bank plans on selling the loan after 2 months. If the required nominal APR increases by 45 basis points when the loan is sold, what loss does the bank incur?

ANSWER: Use the formula for a amortizing loan to determine that the monthly payment is $36,398.28. After two months, the balance is again determined by the amortizing bond formula

\[
P_b = \left( \frac{36398.28}{.08/12} \right) \left( 1 - \left( \frac{1}{1 + .08/12} \right)^{118} \right) = 2,967,094.26
\]

If the interest rate goes to 8.45%, then the price falls to: $2,910,552.15 for a loss of $2967094.26-$2910552.15 = $56,542.09

20. Assume the bank in the previous question partially hedges the mortgage by selling three 10-year T-note futures contracts at a price of 100 32/32. Each contract is for $1,000,000. After 2 months, the futures contract has fallen in price to 98 34/32. What was the gain or loss on the futures transaction?

ANSWER: on each contract the gain is: \((100 32/32 - 98 34/32) \times 10,000 = $18,750\) (the price difference is multiplied by 10,000 since $100 \times 10,000 = 1,000,000 i.e. the value of the contract. So the price is given in (per $100 units).) Multiplying this by 3 yields a total gain of $56,250 so the hedge works well.

30. North-Northwest Bank has a differential advantage in issuing variable-rate mortgages, but does not want to interest income risk associated with such loans. The bank currently has a portfolio of $25,000,000 mortgages with an APR of prime + 150 basis points, reset monthly. Prime is currently 4%. An investment bank has arranged for NNWB to swap into a fixed interest payment of 6.5% on a notional amount of $25,000,000 in return for its variable interest income. If NNWB agrees to this, what interest is received and given in the first month? What if prime suddenly increased 200 basis points?

ANSWER: NNWB earns $25,000,000 \times (0.065/12) = $114,583.33 on the mortgage (which is now going to the investment bank) but is slated to receive $25,000,000 \times (0.065/12) = $135,416.67 so the investment bank gives to NNWB the
difference of $20,833.34. If the prime rate goes up to 6%, then NNWB earns $156,250 so now NNWB gives to the
investment bank: 20,833.33. Note that NNWB is now receiving a constant $135,416.67 despite movements in the prime
rate.
Graphs associated for Q1

ROA
Assets less than 300 million

Assets greater than 15 billion
ROE

Less than 300 million

Assets Greater than 15 Billion
NIM

Assets less than 300 million

Assets Greater than $15 billion