1 Homework #4 Answer Key

1. The Pareto optimum is achieved by maximizing expected utility subject to the resource constraint:

$$\max E[U(c)] = \pi_1 U(c_1) + \pi_2 \rho U(c_2)$$

subject to: $$1 = \pi_1 c_1 + \pi_2 \frac{c_2}{R}$$

Solving for $c_2$ using the resource constraint and replacing this in the expected utility function then taking the derivative with respect to $c_1$ yields:

$$U'(c_1) = \rho U'(c_2) R$$

The utility function exhibits constant relative risk aversion so it is of the form $\frac{1}{\gamma} c^{-\gamma}$ (with $\gamma = 2$). Using this with the other numbers provided in the question yields the following two equations in the two unknowns:

necessary condition: $$c_1^{-2} = (0.5) c_2^{-2}$$

resource constraint: $$1 = \frac{1}{2} c_1 + \frac{1}{2} c_2$$

Solving for consumption yields $c_1 = c_2 = \frac{4}{3}$. Recall that in the market economy, since $R = (1 + r)$ (why??), this implies $c_1^M = 1, c_2^M = R = 2$. In class we discussed that the optimal level of consumption would imply more consumption of type 1 and less of type 2 - precisely what we get.

2. The existence of banks can produce a Pareto optimal allocation since the banks can offer full and fairly priced liquidity insurance. Let $N$ be the number of households where $N$ is large. Each household has the probability of $\pi_1$ that they are early consumers; if they are early consumers, their optimal consumption is $C_1$. This implies that the expected amount of consumption in the first period is $\pi_1 C_1$. With $N$ very large, the amount of consumption in the first period is exactly equal to $\pi_1 C_1$. This is an implication of the Law of Large Numbers.

3. As shown in the answer to Q1 above, the necessary condition associated with the Pareto optimum is:

$$U'(c_1) = \rho U'(c_2) R$$

or, assuming constant relative risk aversion utility

$$c_1^{-\gamma} = \rho c_2^{-\gamma} R$$

We know that in the market equilibrium we have: $c_1^M = 1, c_2^M = R$. If the market equilibrium is also the Pareto optimum then it must satisfy the associated necessary condition. But this is not the case since:

$$(c_1^M)^{-\gamma} = 1^{-\gamma} > \rho (c_2^M)^{-\gamma} R = \rho R^{1-\gamma}$$

The inequality holds because of the assumptions that $\rho < 1$, $R > 1$ and $\gamma > 1$. 