Through DGAP and FGAP, a bank can assess its interest rate risk.

Now we discuss how to reduce this exposure by using financial futures.

First I present an overview of futures markets and the determination of futures prices.
Futures - Overview

The two rules for successful investing

1. Buy Low, Sell High.
2. Be diversified.

Futures contracts are related to both concepts.

First – note that diversification works

1. ONLY if returns are not correlated.
2. Works “best” if returns are negatively correlated.
Futures - Overview

Buy Low, Sell High – usually implies that you must first purchase the item BEFORE you can sell it.

Futures contracts break this temporal arrangement:
If you sell a futures contract: You agree to sell something today at a future date, T, at a set price, $F_t$. If the price of the futures contract falls below that price, then you make money.

AND, the price of the futures contract and the price of the good or asset (the spot price), are usually highly correlated. By taking the correct position in the futures market, you can create a hedged position.
First, some notation. Let

\[ S_t \] denote the current price of the good – the Spot Price.

\[ F_t \] denote the price of the futures contract.

*Basis*: the difference between spot and futures price:

\[ \text{basis}_t = S_t - F_t \]

*Long position*: BUY a futures contract.

*Short position*: SELL a futures contract.

*Delivery Month*: the month the futures contract expires.
Futures Overview

Important to note: futures contracts are traded on organized exchanges:
CBOT – Chicago Board of Trade
IMM – International Monetary Market (a division of the Chicago Mercantile Exchange).
NYMEX – New York Mercantile Exchange

Positions are taken with the Exchange – you can clear your account by taking an offsetting, i.e. reverse, position with the exchange.

If initially Long in pork bellies, go Short on the same contract and you have eliminated your position. Only 2% of contracts take delivery.
Futures Overview

The position you take depends on your forecast of prices:

1. Forecast prices to rise: Buy Low, Sell High
   Take a Long Position.

2. Forecast prices to fall: Sell High, Buy Low
   Take a Short Position

For hedging the same idea: if you are afraid of a fall in prices, take a position to make money if that occurs – i.e. go short.
If concerned about rising prices, take a long position.
By purchasing a futures contract, you substitute price risk with basis risk.

Example: Delta Airlines knows it need to buy jet fuel next month. It is concerned about a price rise. Take a position in the futures market to make money if the worse case scenario occurs:

   GO LONG!
The price paid in the future is given by the spot price and the change in the price of the futures contract:

\[ P_T = S_T + (F_t - F_T) \]

\[ P_T = S_t + [(S_T - F_T) - (S_t - F_t)] \]

\[ P_T = S_t + \Delta \text{basis} (t, T) \]

The relevant price at the time of purchase is equal to today’s spot price plus any change in the basis. The basis is easier to predict than the actual spot price. Why? — the futures price reflects storage and interest costs.
Example: Cash and Carry Gold Arbitrage Transaction

Suppose $S_t = $400 and $F_t = $450 for delivery in one year. The current interest rate is $i = 10\%$.

Arbitrage opportunity: borrow money today to buy gold. At the same time take a short position in the futures market. In one year, deliver gold for futures contract, take money and repay the loan.
### Futures Overview

**Example: Cash and Carry Gold Arbitrage Transaction**

#### Time = 0

<table>
<thead>
<tr>
<th>Transaction</th>
<th>Cash Flow</th>
</tr>
</thead>
<tbody>
<tr>
<td>Borrow $400 at 10%</td>
<td>+$400</td>
</tr>
<tr>
<td>Buy 1oz of Au</td>
<td>-$400</td>
</tr>
<tr>
<td>Sell a futures contract for $450</td>
<td>0</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>0</strong></td>
</tr>
</tbody>
</table>

#### Time = 1

<table>
<thead>
<tr>
<th>Transaction</th>
<th>Cash Flow</th>
</tr>
</thead>
<tbody>
<tr>
<td>Remove Au from storage</td>
<td>$0</td>
</tr>
<tr>
<td>Deliver 1oz Au for contract</td>
<td>+$450</td>
</tr>
<tr>
<td>Repay Loan + interest</td>
<td>-$440</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>$10</strong></td>
</tr>
</tbody>
</table>

Note that the basis in this example is $50
Example: Reverse Cash and Carry Gold Arbitrage Transaction

Suppose \( S_t = \$420 \) and \( F_t = \$450 \) for delivery in one year. The current interest rate is \( i = 10\% \).

Arbitrage opportunity: Borrow gold today, sell it and lend the money. At the same time take a long position in the futures market. In one year, get the repayment of the loan, use it to buy gold (take delivery on futures contract) and repay the gold loan.
### Futures overview

**Example: Reverse Cash and Carry Gold Arbitrage Transaction**

#### Time = 0

<table>
<thead>
<tr>
<th>Transaction</th>
<th>Cash Flow</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sell 1 oz. of Au short</td>
<td>+$420</td>
</tr>
<tr>
<td>Lend the $420 for one year</td>
<td>-$420</td>
</tr>
<tr>
<td>Buy futures contract for $450</td>
<td>0</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>0</td>
</tr>
</tbody>
</table>

#### Time = 1

<table>
<thead>
<tr>
<th>Transaction</th>
<th>Cash Flow</th>
</tr>
</thead>
<tbody>
<tr>
<td>Collect proceeds from loan</td>
<td>$462</td>
</tr>
<tr>
<td>Accept delivery on futures contract</td>
<td>-$450</td>
</tr>
<tr>
<td>Repay Au loan</td>
<td>$0</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>$12</strong></td>
</tr>
</tbody>
</table>

Note that the basis in this example is $30
Futures overview

The Basis reflects storage costs and interest rates – since these are (somewhat) predictable, basis risk is lower than spot price risk.

Now we turn to the kind of futures we are interested in: Financial Futures.
In January 1976, the International Monetary Market (IMM), now part of the Chicago Mercantile Exchange (CME), began trading futures contracts in 13-week Treasury bills.

The basic contract is for $1 million with contracts maturing once each quarter in the third week of March, June, September and December. Since there are eight contracts outstanding, the most distant delivery date varies between 21 and 24 months into the future.
### T-Bill Futures

<table>
<thead>
<tr>
<th>Year</th>
<th>Quarter</th>
<th>Bid</th>
</tr>
</thead>
<tbody>
<tr>
<td>2009</td>
<td>September</td>
<td>10.49</td>
</tr>
<tr>
<td></td>
<td>December</td>
<td>10.85</td>
</tr>
<tr>
<td>2010</td>
<td>March</td>
<td>11.13</td>
</tr>
<tr>
<td></td>
<td>June</td>
<td>11.35</td>
</tr>
<tr>
<td></td>
<td>September</td>
<td>11.52</td>
</tr>
<tr>
<td></td>
<td>December</td>
<td>11.66</td>
</tr>
</tbody>
</table>

### T-Bill Spot

<table>
<thead>
<tr>
<th>Maturity Date</th>
<th>Bid</th>
<th>Asked</th>
</tr>
</thead>
<tbody>
<tr>
<td>August 9, 2009</td>
<td>9,91</td>
<td>9,79</td>
</tr>
<tr>
<td>September 20, 2009</td>
<td>9,95</td>
<td>9.91</td>
</tr>
<tr>
<td>December 20, 2009</td>
<td>10.45</td>
<td>10.39</td>
</tr>
<tr>
<td>March 21, 2010</td>
<td>10.63</td>
<td>10.56</td>
</tr>
</tbody>
</table>

Bid = buy the T-bill from you, Asked – sell the T-bill to you.
T-Bill Futures -

Suppose on August 7 an investor wants to save money to use for Christmas. Two options.

1. Buy T-Bill that matures on Dec. 20 – yield (asked) is 10.39. This is 134 days away. Price per $100 is

\[ P_{T-bill} = \frac{100}{(1.1039)^{134/360}} = 96.41 \]

2. Buy a futures contract that allows him to buy a T-Bill in September that will mature in 3rd week of December. Price to be paid in Sept.

\[ P_{Sept.} = \frac{100}{(1.1049)^{90/360}} = 97.54 \]

Present discounted value of that (discounted to Aug. 7):

\[ P_{Aug} = \frac{97.54}{(1.0991)^{43/360}} = 96.44 \]

Price of both strategies roughly the same!! Arbitrage implies this
Hedging interest rate risk using financial futures

To hedge interest rate risk, take a position in the futures market that makes money if the worst case scenario occurs.
Micro Hedge Example

1. You are the CFO of a small firm that plans to borrow $1 million next year to expand operations. What position should you take in the financial futures market to hedge this action?

Answer: You will be hurt if interest rates rise, or, stated alternatively, bond prices fall. So you want to make money in the financial futures market if this happens:

Take a short position (sell today at the high price).
Hedging interest rate risk using financial futures

Macro Hedge Example

1. You are the CFO of an pension company that has a negative DGAP. What position should you take in the financial futures market to hedge the implied interest rate risk?

Answer: Net worth will be hurt if interest rates fall, or, stated alternatively, bond prices increase. So you want to make money in the financial futures market if this happens:
Take a long position (buy today at the low price).