1. (15) Answer the following in the context of the financial futures market as discussed in class:

(a) A pension fund currently has a negative duration gap. If it wants to minimize its exposure to interest rate risk, what position should it take in the financial futures market?

ANSWER: A negative DGAP implies that the company will be hurt if interest rates fall (why?), or stated alternatively, if bond prices rise. So, to make money if this occurs, the pension fund should take a LONG position: buy now at the low price and sell back at the high price if interest rates do fall.

(b) If a bank has a negative funding gap, what position should it take in the financial futures market in order to minimize its exposure to interest rate risk.

ANSWER: A negative funding gap implies that the bank’s net interest income will be hurt if interest rates rise (why?), or stated alternatively, if bond prices fall. So, to make money if this occurs, the bank should take a SHORT position: sell now at the high price and buy back at the low price if interest rates do rise.

(c) Explain briefly why hedging in the futures market exchanges price risk for basis risk. Why are participants willing to make this exchange?

ANSWER: Let $P_T$ equal the price paid at time $T$ (the terminal date); then this is equal to $P_T = S_T + (F_t - F_T)$ where $S_T$ is the spot price at time $T$, $F_t$, and $F_T$ are the futures price at today and at the terminal date. Add and subtract the current spot price and rearrange term: $P_T = S_t + (S_T - F_T) - (S_t - F_t)$. This shows that the price paid at time $T$ is equal to today’s spot price adjusted for any change in the basis. Agents are willing to accept basis risk since the basis is easier to predict that the spot price. This is the case because the basis depends on storage and interest rate costs. But if the item being hedged is not the same as the item traded in the futures market, then the basis risk is higher.

2. (20) Suppose Corp. A can borrow long term at a fixed rate of 10% or at a floating rate of 50 bp over LIBOR. Corp B can borrow long term at a fixed rate of 11% and a floating rate of 75 bp over LIBOR. If Corp. A desires a floating rate and Corp. B wants a fixed rate, design an interest rate swap that reduces the borrowing costs of both firms.

ANSWER: Corp A has a 1% advantage in fixed rate debt but only a 15bp advantage in floating rate debt. So it is possible to construct a swap in which A issues fixed rate debt while B issues floating rate debt; then A pays a floating rate payment to B and B pays a fixed rate payment to A. Let the floating rate that A pays be equal to Libor (L) and let the fixed rate payment be denoted by $SR$. Then the all-in-costs for A are: 10% (fixed rate debt) + L (floating rate in swap) - SR (received from B). If this is less than the floating rate that A can get on its own, then you have a deal: $10 + L - SR < L + 0.50$ or $9.50 < SR$. We require the analogous condition for B: $L + 0.75 - L + SR < 11$ or $SR < 10.25$. So any rate between 9.50 and 10.25 will be acceptable to both parties.

3. (16) Define the following:

(a) Mortgage pass through bond.

ANSWER: A bond constructed from bundling a pool of mortgages. The mortgage payments are "passed-through" in the form of interest rate payments to the bond holders.

(b) Commercial paper.

ANSWER: A short term debt instrument (maximum maturity is 270 days) issued by large corporations to finance their operations. It is an unsecured loan (i.e. there is no collateral).

(c) Banker’s acceptance.

ANSWER: A short term debt instrument typically used in international trade. A bank "accepts" the debt associated with the sale of goods and will pay the notional amount to the bearer of the BA; as a consequence, the BA is negotiable. In essence, the credit worthiness of the importer (typically) is replaced with that of the bank.
(d) Reverse Repo.

ANSWER: A short term loan in which the lender buys securities but also promises to re-sell them in the future. Since the securities act as collateral, this is a secured loan.

4. (25) Nancy and Bertha are faced with the same type of problem. Both can choose between two career choices: a risky job which has a random income or a safe job that has a certain (i.e. guaranteed) income. Nancy and Bertha receive the same income in the safe job but Nancy’s expected level of income is higher than Bertha’s in the risky career choice. Suppose that Nancy and Bertha are indifferent between the safe and risky career choices. What do you conclude? Justify your answer graphically.

ANSWER: It is assumed that both Nancy and Bertha are risk averse. Let

\[ Y = \frac{U''(\bar{Y})}{U'(\bar{Y})} \text{Var}(Y) \]

where \( U''(\bar{Y}) \) is the measure of absolute risk aversion. Hence the information implies that either Nancy is more risk averse than Bertha AND/OR she faces an income prospect that has a higher variance than that presented to Bertha. The graph to illustrate this is the one used in class and in the reading.

5. (25) In analyzing the demand for insurance, a model was presented with the following features: Risk averse agents had wealth equal to \( W \) but faced the probability of a loss of size \( x \) with probability \( p \). Agents could purchase insurance (offered by insurance firms in a perfectly competitive insurance market) with a premium of \( h \) and the choice variable was the amount of coverage to buy, denoted \( y \). Agents choose \( y \) in order to maximize expected utility. Use that model to answer the following questions:

(a) Consider the initial setting in which agents have no insurance. Prove that the slope of the indifference curve evaluated at the no insurance line is different than the ratio of \( (1-p)/p \). What is the implication?

ANSWER: The slope of the indifference curve is found by taking the total differential of expected utility:

\[ dE[U(W)] = p \frac{dU(W_2)}{dW_2} dW_2 + (1-p) \frac{dU(W_1)}{dW_1} dW_1 \]

where \( W_1 \) is wealth in the no loss state and \( W_2 \) is wealth in the loss state (so \( W_1 > W_2 \) if no insurance is purchased). Since \( dE[U(W)] = 0 \) along an indifference curve we have:

\[ \frac{dW_2}{dW_1} = \frac{1-p}{p} \frac{dU(W_1)}{dU(W_2)} \]

Clearly this will not equal \( \frac{(1-p)}{p} \) if \( W_1 \neq W_2 \). Moreover, since \( W_1 > W_2 \) we know \( \frac{(1-p)\frac{dU(W_1)}{dW_1}}{\frac{p dU(W_2)}{dW_2}} < \frac{(1-p)}{p} \) since agents have diminishing marginal utility. At the no insurance point, the slope of the indifference curve is flatter than that of the market line.

(b) Suppose Bob is more risk averse than Jerry. Given the above scenario, what is the prediction for the amount of insurance that Bob purchases relative to that purchased by Jerry?

ANSWER: We demonstrated in class that a risk averse agent will purchase full insurance when the insurance is actuarially fair. So both Jerry and Bob will purchase full insurance.