Midterm Exam

Directions: Answer all questions; point totals for each question are given in parentheses. For full credit, you must provide complete explanations for your answers.

1. (20) What is the duration of a 3-year bond with 10% coupon rate that is currently selling at par? The coupon payments are made annually.

   ANSWER: The coupon payment is $C_t = rF$ where $r = 0.10$. Also, since the bond is selling at par we have $P = F$ and we know that $i = r = 0.10$. Using all this, the duration formula becomes

   $$D = \frac{0.10F}{(1.10)^1} + \frac{0.10F}{(1.10)^2} + \frac{F(1.10)}{(1.10)^3} = 2.735$$

2. (30) A bank has two 3-year commercial loans with a present value of $70 million. The first is a $30 million loan that requires a single payment of $37.8 million in three years, with no payments until then. The second loan is for $40 million. It requires an annual interest payment of $3.6 million. The principal of $40 million is due in three years.

   (a) What is the duration of the bank’s commercial loan portfolio?

   ANSWER: The duration of the $30m loan is 3 since it is a zero coupon bond. The duration of the $40m loan is determined by the formula (note since the bond is selling at par, it is the case that the yield to maturity = the coupon rate = 9%. So the duration of this loan is $D = \frac{3(3.6/1.09) + (3.6/1.09^2) + (43.6/1.09^3)}{40} = 2.76$. So the duration of the loan portfolio is $[30/70 + (40/70)(2.76)] = 2.86$

   (b) What will happen to the value of its portfolio if the general level of interest rates decreases from 8% to 7.0%?

   ANSWER: Using the duration formula: $\frac{\Delta P}{P} = -2.86 \frac{0.01}{1.08} = 0.0265$. So the change in the value is $\Delta P = 0.0265(70) = 1.85$ million. That is a $1.85 million increase in value.

3. (20) Assume that the expectations hypothesis is true for the following two questions:

   (a) Suppose you see the following bond prices: a 1-year $100 zero coupon bond selling for $90.10; a 3-year 10% coupon bond selling for par and a 2-year coupon bond with annual coupon payments of $100 and a face value of $1000 that is selling for $1000. What is the market forecast of the one year rate 2 years from now?

   ANSWER: The information implies the $i_1 = 0.11$, $i_2 = i_3 = 0.10$. The expectations hypothesis implies $i_1 = 3(0.10) - 2(0.10) = 0.10$

   (b) Suppose the current one-year rate is 5% and two year rate is 6%. If the forecast of the one-year rate a year from now (assumed to be accurate) is 7.25%, construct a trading arrangement that is guaranteed to make you money. (Assume the quoted interest rates are for both borrowing and lending.)

   ANSWER: Borrow money for two years and then invest it in a sequence of one year bonds. Suppose I borrow $100, then in two years I have to pay $100(1.06)^2 = 112.36$. But by lending the money I have made $100(1.05)(1.0725) = 112.61$ so a riskless profit of $(112.61 - 112.36) = 0.25$.

4. (30) The Bank of Dixon has assets totaling $180 million with a duration of 5 years and liabilities totaling $160 million with a duration of 2 years. If interest rates increase from 9% by 75 basis points, what is the change in the bank’s net worth? What is the change in the bank’s equity multiplier and what implication does this have?
ANSWER: The duration gap is given by:
\[ D_{gap} = DA - \left( \frac{L}{A} \right) (DL) = 3.22 \]

The change in net worth as a percentage of assets is: \[ \frac{\Delta NW}{A} = -D_{gap} \frac{\Delta i}{1 + i} = -0.0221 \]. So the change in net worth is \( \Delta NW = 180 \times (-0.0189) = $3.98 \) million implying that net worth is now \( 20 - 3.4 = $16.02 \) million. To calculate the change in the equity multiplier, first note that the original equity multiplier is \( (A/E) = (180/20) = 9 \). We know that equity (i.e. net worth) is now $16.02 million. Since \( DA = 5 \), this implies that the change in assets is equal to \( \Delta A = DA (\Delta i/(1 + i)) A = -6.19 \) so new equity is $173.81 million. The new equity multiplier is therefore \( EM = 173.81/16.6 = 10.85 \). The equity multiplier has increased because the fall in net worth is greater than the fall in assets (in percentage terms). The firm is now in a riskier position (less capital relative to assets).

5. (10) Use the Fisher relation to prove that if actual inflation rates are less than what was expected, then the ex-post real rate is greater than the ex-ante real rate.

ANSWER: Let \( r^{ep} \) denote the ex-post real interest rate and \( r \) denote the ex-ante. We have \( r = i - \pi^e \) where \( i \) is the nominal interest rate and we have \( r^{ep} = i - \pi \). Then \( r^{ep} - r = i - \pi -(i - \pi^e) = \pi^e - \pi > 0 \) (as stated in the problem).

6. (10) When hired as the manager of the Bank of Davis you were given explicit instructions that net interest margin should not vary more than 10% from the current level of 5%. If you think interest rates will change by at most 200 basis points in the next year, how large can your funding gap be (as a percentage of assets) in order for you to keep your job?

ANSWER: We start with the condition that \( \Delta NII = FGAP \Delta i \). Divide both sides assets to yield \( \frac{\Delta NII}{A} = \Delta NIM = \frac{FGAP}{A} \Delta i \). Then multiply and divide the left hand side to get the % change in NIM: \( \% \Delta NIM \ NIM = \left( \frac{FGAP}{A} \right) \Delta i \). Or \( \frac{FGAP}{A} = \% \Delta NIM \ NIM = \frac{\Delta (0.10)(0.05)}{0.02} = 0.25 \) or the FGAP should be at most 25% of assets.