Recent Developments in Modeling Financial Intermediation
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Four equations define the model:

\begin{align*}
    s_t &= e - \frac{p_t}{E_t(p_{t+1})} \\
    \pi_t^* w_t - (1 - \pi_t^*) \beta &= r_t K \\
    L_t &= (1 - \alpha) NK \frac{(\pi_t^u - \pi_t^*)}{(\pi_t^u - \pi_t^l)} \\
    \alpha N s_t &= p_t H + L_t
\end{align*}
Exogenous Uncertainty

- A two-state Markov process with \((z_1, z_2)\) and
  \[ \Pr(z_{t+1} = z1|z_t = z_i) = q_i. \]
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- Possible realizations:

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  z_1 = \left( \pi^l_1 = 0.1, \pi^u_1 = 0.9, w_1 = 400 \right) \\
  z_2 = \left( \pi^l_2 = 0.08, \pi^u_2 = 0.7, w_2 = 514 \right)
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But there is change in riskiness of projects.
State 2 implies greater risk relative to State 1

- Consider a project whose expected returns are the same in both states. For example

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So in state 2 we also want to keep \( R_2 = R_1 = 200 \). Since \( w_2 = 514 \), this requires

\[ R_2 = 200 = E_2(w) = \pi_2(514) \Rightarrow \pi_2 = \frac{200}{514} = 0.39 \]
State 2 implies greater risk relative to State 1

But consider the variance:

\[
\begin{align*}
\text{Var}_i (R) &= \text{Var}_i (w) = E_i (w^2) - [E_i (w)]^2 \\
&= \left[ \pi_i (w_i)^2 - (1 - \pi_i) 0^2 \right] - R_i^2 \\
\text{Var}_i (R) &= \pi_i w_i - R_i^2 \text{ (since } \pi_i w_i = R_i) \\
\end{align*}
\]

Since \( R_1 = R_2 \) by assumption, then \( \text{Var}_2 (R) > \text{Var}_1 (R) \) since \( w_2 > w_1 \).
By assumption \( \Pr (z_{t+1} = z_1 | z_t = z_1) > \Pr (z_{t+1} = z_1 | z_t = z_2) \)
Equilibrium

- By assumption $\Pr(z_{t+1} = z_1 | z_t = z_1) > \Pr(z_{t+1} = z_1 | z_t = z_2)$
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- So, even though there is no change in the average level of returns, output will be less following state 2 relative to state 1.
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Endogenous business cycles because of financial intermediation.