Choice Under Uncertainty

• So far, all decisions have taken place in a world of certainty
  ■ consumers know for sure the utility they will receive given a choice of goods
  ■ firms know for sure the profit they will receive from a chosen set of inputs

• This does not describe the real world
  ■ technological uncertainty
  ■ market uncertainty

• Many issues cannot be addressed without considering uncertainty eg. stock market, insurance, futures markets (investment and savings decisions)
• The next few lectures will proceed as follows
  ■ attitudes towards risk
  ■ uncertainty in the insurance market
Attitudes towards Risk

- Consider a representative agent model in which there is a single good (income).

- Secondly, assume that there are only two states of the world (state 1 and state 2).
  - Let $y_1$ denote income in state 1 and $y_2$ income in state 2.
  - Let $\pi$ denote the probability of state 1 so that $1 - \pi$ is the probability of state 2.

- Given this, the expected value of income is

$$\bar{y} = \pi y_1 + (1 - \pi) y_2$$

- the weighted average level of income received by the individual over the two states of the world.
• Utility of the expected value is

\[ u(\bar{y}) = u(\pi y_1 + (1 - \pi)y_2) \]

• the utility received by the individual from the weighted average level of income \( \bar{y} \)

• Expected utility is given by

\[ \bar{u} = \pi u(y_1) + (1 - \pi)u(y_2) \]

• the utility received by the individual from fluctuating income levels across the two states of the world
Risk Aversion

- An individual is risk averse if
  \[ u(y) > u \]

- He/she prefers to have a constant amount of income rather than fluctuating amounts of a low income in state 1 of the world and a high income in state 2 of the world
  - averages are preferred to extremes

- An alternative definition uses the **certainty equivalent level of income** denoted by \( y_c \)

- This level of income must satisfy the following condition
  \[ u(y_c) = u \]
  - \( y_c \) must be such that the utility received from getting \( y_c \) equals the utility from facing the gamble (expected utility)
• In other words, \( y_c \) is the level of income at which the individual is indifferent between getting that level of income for certain and facing fluctuating incomes levels.

• This certainty equivalent level of income therefore gives us a notion of the value of the gamble to the individual. If the individual is risk averse

\[
\overline{y} > y_c
\]

that is, the individual values the gamble at less than the expected value.
Risk Neutral

- An individual is risk neutral if he/she values the prospect at its’ expected value

\[ \overline{y} = y_c \]

or

\[ u(\pi y_1 + (1 - \pi)y_2) = \pi u(y_1) + (1 - \pi)u(y_2) \]

Risk Loving

- An individual is loving if he/she values the prospect at more than its’ expected value

\[ y_c > \overline{y} \]

or

\[ \pi u(y_1) + (1 - \pi)u(y_2) > u(\pi y_1 + (1 - \pi)y_2) \]
The Risk Premium

- Consider figure 1 and recall that \( y_c \) satisfies 
  \[ u(y_c) = u \]

- In figure 1, the individual is willing to give up income \( cd \) rather than face the gamble with expected income \( \bar{y} \)
  \[ r = \bar{y} - y_c > 0 \]

- If the individual is risk loving, the individual would pay to be able to face the gamble
  \[ r = \bar{y} - y_c > 0 \]
Measures of Risk Aversion

- How risk averse is a particular individual?
- The more concave the utility function, the more risk averse the individual.
• Formally, risk aversion can be measured by

\[ A(y) = -\frac{u''(y)}{u'(y)} \]

This is the Arrow-Pratt coefficient of absolute risk aversion

• If the individual is risk averse, \( u''(y) < 0 \) which implies \( A(y) > 0 \)

• \( A(y) \) may increase, decrease or remain constant as \( y \) increases
  
  ■ If the risk premium is decreasing (increasing) in wealth, the consumer has decreasing (increasing) risk aversion
  
  ■ If the risk premium is constant in wealth, the consumer has constant risk aversion
Insurance Under Uncertainty

• Consider a risk averse individual and suppose that there are two states of the world: state 1 and state 2
  ■ state 1 - initial income is \( y \)
  ■ state 2 - income is \( y - L \)

• The consumer can insure against the loss (cannot affect the loss or the probability of the loss ⇒ moral hazard issue)

• The insurance company sells insurance at a premium rate \( p \) (\( 0 < p < 1 \)). Let \( q \) denote the amount of insurance cover purchased by the consumer

\[
y_1 = y - pq \\
y_2 = y - L - pq + q
\]
• Let \( \pi_1 \) denote the probability of state 1 and let \( \pi_2 \) denote the probability of state 2

\[
\bar{y} = \pi_1(y - pq) + \pi_2(y - L - pq + q)
\]

• Now suppose that the individual maximises expected utility

\[
\max_q \pi_1 u(y_1) + \pi_2 u(y_2) = \max_q \pi_1 u(y - pq) + \pi_2 u(y - L - pq + q)
\]

• Rearranging the first order condition gives the following familiar condition

\[
\frac{u'(y_2)\pi_2}{u'(y_1)\pi_1} = \frac{p}{1 - p}
\]
• An insurance company has an actuarially fair premium \( p \) if it does not alter the insured individual’s expected income

\[
\pi_1(y - pq) + \pi_2(y - L - pq + q)
\]
\[
= \pi_1 y + \pi_2 (y - L)
\]

• Hence, the fair premium is \( p = \pi_2 \)

• In fact, perfect competition implies a fair premium

• Competition in the industry forces the expected profits of the firm to zero

\[
\pi_1 pq + \pi_2 (pq - q) = 0
\]
\[
\Rightarrow
\]
\[
q(p - \pi_2) = 0
\]
Case 1

- Suppose $p = \pi_2$ (premium is fair). Then, from the foc, we can write

$$u'(y_1) = u'(y_2)$$

- Since $u''(y) < 0$, this implies $y_1 = y_2$

$$y - pq* = y - L + (1 - p)q^*$$

$$\Rightarrow$$

$$L = q^*$$

- the insurer completely insures against the loss