Assessing Interest Rate Risk:
The Funding Gap and Duration Gap Analysis

We focus on ways to measure interest rate risk exposure of banks. There are two main views:
1. Funding Gap: this is the income view. How will interest rate fluctuations affect NII?
2. Duration Gap: this is the balance sheet view. How will interest rate fluctuations affect equity?

Conclusion: both are important. Can not insulate both risks simultaneously.

Funding Gap – Managing Interest Income in the short term

Implementation is simple:
1. Divide up assets/liabilities into maturity buckets
2. Within each bucket, determine the value of rate sensitive assets (RSA) and value of rate sensitive liabilities (RSL).
3. Then the funding gap is simply:

\[ \text{FGAP} = \text{Value of RSA} - \text{Value of RSL} \]

Why is this useful? Recall definition of NII:

\[ NII = FRA \cdot r_f + RSA \cdot r_s - FRL \cdot i_f - RSL \cdot i_s \]

Where FRA = fixed rate assets, FRL = fixed rate liabilities. Then, the change in NII can be written as (where I assume \( \Delta r_f = \Delta r_s = \Delta i_f = \Delta i_s \)):

\[ \Delta NII = RSA \cdot \Delta r_s - RSL \cdot \Delta i_s = (RSA - RSL) \Delta i = \text{FGAP} \cdot \Delta i \]

Note that \( \text{Sign} [\text{Cov}(\Delta NII, \Delta i)] = \text{Sign}[\text{FGAP}] \)

Funding Gap – Example

Consider the following simple example:

<table>
<thead>
<tr>
<th>Balance Sheet of Southern Rock Bank</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td><strong>Assets</strong></td>
</tr>
<tr>
<td>Rate Sensitive</td>
</tr>
<tr>
<td>Fixed Rate</td>
</tr>
<tr>
<td>Equity</td>
</tr>
<tr>
<td><strong>Total</strong></td>
</tr>
</tbody>
</table>

\( NII = 40.90, \quad NIM = 4.81\%, \quad \text{FGAP} = -100. \)

Suppose \( \chi_i = +1\% \) then \( NII = 39.90. \)
Funding Gap and NIM variation

The relationship between FGAP and NIM can be used to accept an acceptable variation in NIM for a given forecast in interest rate variation. Note that:

$$FGAP \cdot \Delta t = \frac{\Delta NI}{N\cdot I} \cdot (\text{Assets}) = \left( \frac{\Delta NI}{N\cdot I} \right) \cdot (\text{Assets})$$

Rewrite this as:

$$\frac{\text{Target Gap}}{\text{Assets}} = \left( \frac{\Delta NI}{N\cdot I} \right)$$

Funding Gap – Managing Interest Income in the short term

Suppose a bank has $50 million in assets and wants a NIM of 5%. It is willing to accept a variation of NIM of 20%. If it forecasts that interest rate volatility of 4% is possible, what is the acceptable FGAP?

Given assets of $50 million, this implies a FGAP = 12.5 million (could be positive or negative).

Duration Gap

Duration Gap is the difference between the average duration of assets and the average duration of liabilities.

<table>
<thead>
<tr>
<th>Main Street Bank’s Assets</th>
<th>Value</th>
<th>Rate</th>
<th>Duration</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cash</td>
<td>500</td>
<td>0%</td>
<td>0</td>
</tr>
<tr>
<td>8 yr loan</td>
<td>700</td>
<td>14%</td>
<td>2.65</td>
</tr>
<tr>
<td>5 yr T-Bond</td>
<td>200</td>
<td>12%</td>
<td>5.07</td>
</tr>
<tr>
<td>Cash</td>
<td>1000</td>
<td></td>
<td>3.05</td>
</tr>
</tbody>
</table>

$$\bar{D}_A = \sum_{i=1}^{N} \left( \frac{V_i}{T_i} \right) \cdot D_i$$

$$\bar{D}_A = \left( \frac{700}{8} \right) (2.65) + \left( \frac{200}{5} \right) (5.07) = 3.05 \text{ years}$$
Duration Gap is the difference between the average duration of assets and the average duration of liabilities.

### Main Street Bank's Liabilities

<table>
<thead>
<tr>
<th>Value</th>
<th>Rate</th>
<th>Duration</th>
</tr>
</thead>
<tbody>
<tr>
<td>5-yr Time Deposit</td>
<td>5%</td>
<td>1</td>
</tr>
<tr>
<td>3-yr CD</td>
<td>10%</td>
<td>3.49</td>
</tr>
<tr>
<td>Total Liabilities</td>
<td></td>
<td>3.49</td>
</tr>
<tr>
<td>Equity</td>
<td></td>
<td>3.49</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>6.98</td>
</tr>
</tbody>
</table>

\[
D_L = \sum_{i=1}^{N} \left( \frac{1}{1 + i} \right) D_i
\]

\[
D_L = \left( \frac{1}{6.98} \right) (1.00) + \left( \frac{1}{3.49} \right) (3.49) = 1.92 \text{ years}
\]

Duration Gap concerns:

1. Requires calculation of duration of all assets and liabilities – this requires much information: each account’s interest rate information, prepayment options, default probabilities, etc.
2. Need estimates of changes in interest rates of different maturities and risk.
3. Duration changes over time and at different rates for different securities – Duration drift. So in practice it is necessary to recalculate DGAP frequently.

Final topic: Impossible to manage FGAP and DGAP simultaneously

It is not possible to simultaneously immunize against interest rate risk as implied by FGAP and DGAP.

To demonstrate this, first note that the equity of a firm is given by the present discounted value of dividends. If a firm is infinitely lived and pays a constant fraction \( k \) of NII as dividends, then equity is given by the price of a consol (note that NII is written as a function of interest rates)

\[
E = \frac{E \times NII(i)}{1}
\]

Take the derivative with respect to interest rates:

\[
\frac{dE}{di} = \frac{\frac{dNII(i)}{di} \times E - \frac{E \times dNII(i)}{di}}{dE} = \frac{\frac{dNII(i)}{di} \times E - \frac{E \times dNII(i)}{di}}{dE}
\]

Express this in discrete change form, i.e., \( E \approx \Delta E \)

\[
\Delta E = \left( \frac{dNII(i)}{di} - \frac{E \times dNII(i)}{di} \right) \Delta i
\]
Final topic: Impossible to manage FGAP and DGAP simultaneously

Rewriting:

\[ \Delta F = \frac{\Delta \text{NPV} - \Delta \Pi}{\Delta \Pi} \]

Recall that \( \Delta \text{NPV} = \Delta \text{FGAP} \cdot \Delta \pi \).

So, if \( \text{FGAP} = 0 \), then equity will fall (due to increase in discount factor).

To immunize equity, then \( \text{FGAP} \) must be positive.

For practical purposes, probably best to focus on equity (DGAP).

Hedging Interest Rate Risk

This quick overview presented some quantitative measures of banks’ exposure to interest rate risk.

The fact is that most banks (of financial intermediaries) will have, by the very nature of their business, a positive DGAP.

What to do?

Hedge interest rate risk using interest rate futures.