Second Midterm Exam

Directions: Answer all questions - they are weighted equally. Remember, to receive full credit you must provide complete explanations for your answers. Relax and Good Luck.

1. Within the context of the Lucas Imperfect Information model, answer the following

(a) It was assumed that agents in the model formed expectations rationally. This implied that they knew that the aggregate price level, \( p \), was distributed normally with mean of \( \mu_p \) and variance of \( \sigma_p^2 \) while the relative price of good \( i \), \( r_i \), was distributed normally with mean of 0 and variance of \( \sigma_i^2 \). Furthermore, it was known that \( p \) and \( r_i \) were independently distributed. Derive the implied mean and variance of the price of good \( i \), that is, \( p_i \).

(b) At the core of the model is a signal extraction problem. Describe the nature of this problem and how its solution determines the slope of the aggregate supply curve.

(c) What are the exogenous sources of uncertainty in the model?

Answer:

a. We have \( p \sim N(\mu_p, \sigma_p^2), r_i \sim N(0, \sigma_i^2) \). Also \( p_i = p + r_i \). Then 
\[
E(p_i) = E[p + r_i] = \mu_p.
\]

b. The signal extraction problem is to extract the signal of the relative price shock \( r_i \) out of the observed movement in the price of your own price \( p_i \). If agents knew that the movement in \( p_i \) was due entirely to \( r_i \) then they would work more (and output would go up). If they knew that the movement in \( p_i \) was entirely due to inflation, they would not increase their work effort. Since they don’t know exactly, they take an average. The component of \( r_i \) they attribute to the \( p_i \) (i.e. the signal that they extract) depends on the relative variances of \( \sigma_p^2 \) and \( \sigma_i^2 \). If \( \sigma_p^2 >> \sigma_i^2 \), then they will infer that most of the change in their own price is due to inflation. The implication for the Phillips curve is that the slope will also be determined by the relative variances of the shocks hitting the economy. Comparing two economies that have similar variances of idiosyncratic shocks but with different volatility of money shocks, the economy with the greater variance of nominal shocks will have a steeper Phillips curve.
c. The exogenous sources of the model are money shocks and idiosyncratic shocks to the demand curves for each good.

2. When Lucas won the Nobel prize, the press release stated that, “Lucas was able to explain why the Phillips curve appeared to have so much empirical support, he could also show that any attempt to exploit the Phillips curve and permanently increase employment by systematically creating higher inflation would be futile.” What does this mean?

**Answer:** Lucas demonstrated that equilibrium in his model produces a positive relationship between inflation and output. That is, the model produces a Phillips curve in the data. But if policy makers attempt stimulate output by increasing the average growth rate of money (Note: monetary policy is characterized by the distribution of the money supply, not by a particular realization of the money stock.) then the Phillips curve will shift reflecting the higher expected inflation rate implied by agents’ rational expectations.

3. Again quoting for the press release announcing Lucas’s Nobel prize: “The fate of the Phillips curve demonstrates the dangers in uncritically using a statistical relationship to draw economic-policy conclusions. The insights into these dangers were developed further in the so-called Lucas Critique.” Explain what this means; in particular, discuss the insights that were developed in the Lucas Critique.

**Answer:** The Lucas critique is simple: when the rules of the game change, agents behavior will change (typically). In the context of the Phillips curve, this is reflected that a change in monetary policy does not result in a movement along the curve but a shift in the curve. The general insights is that it is very difficult to use responses estimated under one policy regime to predict the response to a change in regime. The estimated behavior capture the rules that were in place at the time. If the policy changes, those estimated responses are no longer valid. Lucas demonstrated this in the context of the investment tax credit.

4. The following questions are drawn from the Lucas Critique model:
   
   **(a)** In deriving the investment demand function associated with a hypothetical industry, Lucas derived the following necessary condition:
   
   \[
   (1 - \psi_t) = \left( \frac{1}{1 + \tau} \right) \{ \lambda (1 - \theta) E_t (p_{t+1}) + \theta \delta + (1 - \delta) [1 - E_t (\psi_{t+1})] \}
   \]

   Interpret this expression in terms of marginal cost = marginal benefit; identify the role of each term in the expression.
(b) Lucas criticizes Hall and Jorgenson’s econometric analysis of the effects of an investment tax credit on investment demand. What is the nature of Lucas’s criticism? In your discussion, use the above necessary condition to support your argument.

Answer:

\[
\frac{1 - \psi_t}{\text{cost of one unit of capital}} = \left( \frac{1}{1 + \rho} \right) \left\{ \begin{aligned}
\lambda (1 - \theta) E_t (p_{t+1}) + \frac{\theta \delta}{\text{after-tax revenue}} & \quad \text{reduced taxes due to depreciation} \\
+ (1 - \delta) \left[ 1 - E_t (\psi_{t+1}) \right] & \quad \text{value of depreciated capital}
\end{aligned} \right\}
\]

The above expression represents the marginal cost = marginal benefit condition associated with optimal investment. The LHS represents the cost at time \( t \) of investment. As shown by the expression in brackets, greater investment has three effects: it increases after-tax revenue, it lowers costs through depreciation, and the remaining capital can be sold resulting in capital gains or losses. Since neither next period’s price or the tax credit are known at time \( t \), the expected value of these random variables is the relevant term.

b. Hall and Jorgenson’s analysis implicitly assumed that the investment tax credit was permanent. Lucas demonstrated that when the regime changed (to one of the investment tax credit being implemented), if agents forecast this to be a temporary situation (and this makes sense in terms of policy), then the response of capital will be much greater than under the Hall and Jorgenson assumption that it is permanent. In the above expression, Hall and Jorgenson implicitly assumed that \( \psi_t = E_t (\psi_{t+1}) \). That is, the assumed that the tax credit was permanent - this mitigates the expected capital gains on investment if the tax credit is temporary.