Midterm Exam - Answer Key

Directions: Answer all questions; point totals for each question are given in parentheses. For full credit, you must provide complete explanations for your answers.

1. (10) True or False: Federal Government expenditures, as a percentage of GDP, have been growing over time. Support your answer with relevant data and statistics.

**Answer:** False. Total federal government expenditures have averaged roughly 22% over the last 40 years. What has grown over time is expenditures on entitlements (Social Security and Medicare being the largest of these). Defense expenditures have declined over time.

2. (20) In his model on the time inconsistency problem, Stanley Fischer examines optimal tax policy when there is no commitment mechanism. Provide a verbal description of how he solves for the time consistent solution in this setting. What characterizes the optimal level of taxes in the time consistent equilibrium. Why does the time inconsistent solution (which is NOT an equilibrium) produce higher utility?

**Answer:** To find the time consistent solution, Fischer works backwards. In the second period, the government takes as exogenous the capital household’s choice in the first period. It also knows the household’s demand functions for consumption and labor in the second period. Facing these demand functions and treating the capital stock as exogenous, the government chooses tax rates on capital and labor and the level of government spending. Note that all of these are functions of the beginning of period capital. In particular, the tax rate on capital \( \tau_k = f(k) \). Household’s optimize twice: in the last period, they take the capital stock as exogenous along with the tax rates and choose optimal consumption and labor. This optimization problem defines consumption and labor as functions of the capital stock and tax rates. These are used to define the indirect utility function for household’s in the second period. In the first period, agents choose consumption and capital taking as exogenous the tax rates - the utility from capital is determined by the indirect utility function determined in the first optimization problem. Note this defines the choice of capital as a function of the tax rate on capital (the optimal tax rate on labor is zero.) That is \( k = g(\tau_k) \). Equilibrium requires consistency which is a fix-point: \( \tau_k = f(g(\tau_k)) \). The optimal level of taxes in the time consistent solution are characterized by a zero tax rate on labor. This is optimal since capital is inelastically supplied in the second period. The time inconsistent solution produces higher utility since it assumes that households are fooled: they believe the government that it will tax both labor and capital next period. This implies a lower tax rate on capital so households save more. In the second period, the government switches to the optimal tax structure of taxing only capital. Since there is more capital (a larger tax base) the government can provide more government services.
3. (20) President Obama is faced with the Ramsey problem of choosing a sequence
of excise taxes so that the utility of U.S. households is maximized. He uses
this revenue to pay for a new stimulus package that, because of policy lags, will
be implemented in the second period of the economy. The government knows
that since households have logarithmic preferences and that $\beta = 1 / (1 + r)$,
households’ optimal consumption is given by:

$$c^*_t = \frac{\frac{1}{1 + \tau_t} \left( \sum_{t=0}^{\infty} \frac{Y_t}{(1+\gamma)^t} \right)}{1 + \tau_t}$$

(1)

$Y_t$ denotes income and it is assumed that $Y_t = $100,000 in every period. The
government plans to stimulate the economy by making a one-time purchase
of President Obama’s favorite cigarettes (rumored to be American Spirit) of
$110,000 in the second period (that is, $G_1 = $110,000) and nothing in all
other periods; that is, $G_t = 0$ for $t = 0; t \geq 2$. Many politicians are saying
that taxes need to be raised next period in order to cover the entire cost of this
stimulus package. Their argument is that it is unfair to burden future gener-
ations with this one-time expense. You have been invited to give your expert
opinion on the matter and adroitly demonstrate that, given the interest rate of
10%, the optimal tax (note, this is not the tax rate) is simply $10,000 every pe-
riod. Present your proof and demonstrate that it is optimal for $\tau^* = r$. (Hint:
Use the conditions that the tax smoothing hypothesis implies taxes are constant,
$\tau_t = \tau^*$, and the government’s intertemporal budget constraint implies that
the present discounted value of tax revenues (given by $c_t \tau_t$, in every period) equals
the present discounted value of government expenditures. You do not have to
solve for the indirect utility function.)

**Answer:** Given the numbers, optimal consumption is (using $\sum_{t=0}^{\infty} \frac{1}{(1+\gamma)^t} = \frac{1}{1+r}$)

$$c^* = \frac{100,000}{1 + \tau^*}$$

Tax revenues are therefore

$$c^* \tau^* = 100,000 \frac{\tau}{1 + \tau}$$

The present discounted value of tax revenues must be equal to the present dis-
counted value of government expenditures (again using $\sum_{t=0}^{\infty} \frac{1}{(1+\gamma)^t} = \frac{1+r}{r}$)

$$100,000 \frac{\tau^*}{1 + \tau^*} \left( \frac{1.10}{0.10} \right) = \frac{110,000}{1.10}$$

which requires $\tau^* = 0.10$.

4. (20) Do you agree with the following quote: “In a closed economy (i.e. no
international trade), Ricardian equivalence implies that national savings will be
invariant to a deficit financed tax cut.” Be precise in your answer.
**Answer:** I agree. A deficit financed tax cut means, by definition that public (i.e. government) savings has fallen. Let $S_{g,t} = T_t - G_t$ denote government savings which is income minus expenditures. Household savings is defined analogously: $S_{h,t} = Y_t - T_t - C_t$. But, from the aggregate resource constraint $Y_t = C_t + I_t + G_t$ we can rewrite this as $S_{h,t} + S_{g,t} = I_t$. We know that Ricardian equivalence implies that economy activity is not affected by the tax cut so $I_t$ is unchanged. Hence, it must be the case that private and public savings perfectly offset each other.

5. (20) Nordhaus criticizes the Stern Review on the grounds that the Ramsey optimal growth model places restrictions on the choice of households’ subjective discount rate, the curvature of the utility function, the rate of consumption growth and the rate or return on capital (i.e. savings). Do you agree with Nordhaus’s criticism? A good answer will use the necessary condition associated with households’ optimal consumption/savings decision to motivate the analysis.

**Answer:** Yes I agree. The necessary condition associated with consumption/savings is

$$U_{c,t} = \beta U_{c,t+1} (1 + r)$$

where I assume that the return on savings is constant. As shown in class, a common functional form for utility is

$$U(c_t) = \begin{cases} c_t^{1-\alpha} / (1 - \alpha) ; & \alpha > 0, \alpha \neq 1 \\ \ln c_t ; & \alpha = 1 \end{cases}$$

where $\alpha$ measures the curvature of the utility function. Then the necessary condition can be written as

$$\left(\frac{c_{t+1}}{c_t}\right)^{\alpha} = \beta (1 + r)$$

Define $\gamma$ as consumption growth and let $\rho$ be agent’s subjective rate of time preference. Then this becomes:

$$(1 + \gamma)^{\alpha} = \frac{1 + r}{1 + \rho}$$

Taking logs and using the approximation that $\ln (1 + x) \approx x$ we have $\alpha \gamma = r - \rho$. Clearly the four terms are related.

6. (10) Provide the dates (roughly) of three post-World War II recessions. Compare and contrast features of post-WWII and pre-WWI features of the business cycle as detailed in C. Romer’s article.