1. (40) Consider an economy that lasts for two periods. Households in this economy have income in the first period of life denoted $Y_0$; they have no income when old. They can save at the interest rate, $r$. Each period, the household faces excise taxes (denoted $\tau_0, \tau_1$) on consumption purchases. Hence the budget constraint for the household in each period is:

$$
Y_0 = c_0 + c_0 \tau_0 + s_0 \\
s_0 (1 + r) = c_1 + c_1 \tau_1
$$

Where $s_0$ denotes savings in the first period of life. Preferences are given by:

$$
\ln c_0 + \beta \ln c_1
$$

(a) Derive the household’s intertemporal budget constraint.

(b) Form the Lagrangian function associated with the household’s utility maximization problem and derive the associated necessary conditions. Assume that $\beta = (1 + r)^{-1}$. Solve for the demand functions, $c_0^*, c_1^*$.

(c) Derive the household’s indirect utility function.

(d) The government in the economy has expenditures in the first period of $G_0$ and purchases nothing in the second period. The government can borrow at the same interest rate as households, $r$. Derive the government’s intertemporal budget constraint.

(e) The government chooses excise $(\tau_0, \tau_1)$ in order to maximize household’s utility while at the same time satisfying the intertemporal budget constraint. Express this Ramsey problem as a Lagrangian.

(f) Derive the necessary conditions and (again, assume that $\beta = (1 + r)^{-1}$) prove that $\tau_0 = \tau_1 = \tau^*$. Why is it optimal to have taxes constant?
2. (25) The title of Fischer’s (1980) influential paper that we studied in class is, “Dynamic Inconsistency, Cooperation and the Benevolent Dissembling Government.” Explain why the adjectives modifying “Government” are particularly relevant and succinctly summarize the time inconsistency problem. Be precise in your answer.

3. (25) The modern version of the IS curve states that the current output gap, \( x_t \), is affected by three factors: \( i_t \), \( E_t(\pi_{t+1}) \), \( E_t(x_{t+1}) \). Explain the rationale for the presence of these terms; in your answer, be sure to identify how the modern version of the IS curve differs from the original IS specification.

4. (25) In his critique of economic policy analysis, Lucas derived the following demand curve for capital (i.e. investment) in a hypothetical industry:

\[
k_t (1 - \delta) + i_t = \frac{1}{\lambda} E_t(a_{t+1} + b r_t \left(1 - \theta_t + \delta \right) + b \left[ \psi_t (1 + r_t) - E_t(\psi_{t+1}) (1 - \delta) \right]
\]

where \( r_t \) denotes the current one-period interest rate, \( \theta_t \) is the current tax rate on profits, \( \psi_t \) is the investment tax credit. Answer the following:

(a) Explain why the factors on the right-hand side of eq. (1) affect investment demand.

(b) What two properties were used to derive this investment demand function.

(c) Lucas criticized Hall and Jorgenson’s analysis of the 1962 tax credit - where did they go wrong?

5. (25) An implication of optimal monetary policy (as described by Clarida, Gali, and Gertler) is the new policy tradeoff between output and inflation variability. Using graphical analysis, illustrate this tradeoff and demonstrate how policymakers’ preferences toward output and inflation volatility influence the economy.

6. (25) According to L. Meyer, at the end of 1997, the actual unemployment rate had fallen below what members of the FOMC believed was NAIRU. Explain why Chairman Greenspan was reluctant to increase the federal funds rate, while some other committee members were in favor of a raise. Interpret this debate in terms of the Taylor rule.