Inflation Targeting in a Simple New Keynesian Model

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Components of the Model

- Expectations augmented Phillips Curve ($PC$ curve)
- Monetary Policy Rule ($MPR$) derived from policymaker’s preferences

**Equilibrium**

- Short Run: intersection of $PC$ and $MPR$ curves
- Long Run: $\pi^e = \pi^T$ That is: Expected inflation = inflation target

**Results**

- “New Policy Tradeoff” - volatility tradeoff in the economy
- Characterization of optimal policy in terms of an interest rate rule.
The expectations augmented Phillips Curve

The Phillips curve is expressed as:

\[ \pi = \pi^e + ax + e \]

- \( \pi^e \) = expected inflation
- \( x = \frac{y - y^n}{y^n} \) where \( y = GDP \) and \( y^n = \) full employment GDP. Therefore \( x = \% \) deviation from full employment.
- \( a \) = the slope of the Phillips curve
- \( e \) = random shock to inflation (say due to oil prices).

Note that the location of the Phillips curve is determined by inflationary expectations and the shock \( e \).
Policymaker’s preferences

We assume that the policymaker cares about inflation and output volatility:

$$\min \frac{1}{2} \left[ k \left( \pi - \pi^T \right)^2 + \lambda x^2 \right]$$

- The importance of inflation volatility given by $k$. 

We know from the PC that $\pi = \pi^e + ax + e$. We will ignore uncertainty so set $e = 0$. Use this to eliminate $x$. 

Take the derivative with respect to $\pi$ and set $\frac{d}{d\pi} = 0$. 
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$$\min_{\pi} \frac{1}{2} \left[ k \left( \pi - \pi^T \right)^2 + \lambda \left( \frac{\pi - \pi^e}{a} \right)^2 \right]$$
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Monetary Policy Rule

The first-order condition is:

\[ k \left( \pi - \pi^T \right) + \lambda \left( \frac{\pi - \pi^e}{a} \right) \frac{1}{a} = 0 \]

or

\[ k \left( \pi - \pi^T \right) + \frac{\lambda}{a} \left( \frac{\pi - \pi^e}{a} \right) = 0 \]

or

\[ ak \left( \pi - \pi^T \right) = -\lambda x \]

This has standard $MC = MB$ interpretation
Interpreting $MC = MB$ condition

Recall preferences are given by:

$$\min_{\pi} \frac{1}{2} \left[ k \left( \pi - \pi^T \right)^2 + \lambda \pi^2 \right]$$

Suppose that $x < 0$ (below full employment). Then policymakers need to increase output $\Delta x > 0$.

- Then $MB = -\lambda x \Delta x$ (use minus sign so that $MB > 0$).
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- Then \( MB = -\lambda x \Delta x \) (use minus sign so that \( MB > 0 \)).
- But this will create inflation as economy moves along Phillips curve \( MC = k \left( \pi - \pi^T \right) \Delta \pi \).
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- But along the Phillips curve $\Delta \pi = a \Delta x$ so $MC = k \left( \pi - \pi^T \right) a \Delta x$
- Setting $MC = MB$ we have $k \left( \pi - \pi^T \right) a \Delta x = -\lambda x \Delta x$
Monetary Policy Rule

Solve this expression for $x$: $a k (\pi - \pi^T) = -\lambda x$

$x = -\left(\frac{k}{\lambda}\right) a (\pi - \pi^T)$

- If $\pi > \pi^T$ then Fed needs to create a recession ($x < 0$): *Leaning against the Wind*
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$$x = - \left( \frac{k}{\lambda} \right) a \left( \pi - \pi^T \right)$$

- If $\pi > \pi^T$ then Fed needs to create a recession ($x < 0$): *Leaning against the Wind*
- We add shocks to the economy that affect output:
  $$x = - \left( \frac{k}{\lambda} \right) a \left( \pi - \pi^T \right) + u$$
Solve this expression for $x$:

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- We add shocks to the economy that affect output:
  $$x = - \left( \frac{k}{\lambda} \right) a (\pi - \pi^T) + u$$
- Solve for $\pi$

$$\pi = \pi^T - \alpha (x - u)$$

where $\alpha = \left( \frac{\lambda}{k} \right) \frac{1}{a}$. The slope of $MPR$ is determined by the relative importance of output/inflation fluctuations to policymaker.
Suppose inflation is currently above the inflation target - at the point $E_1$ in the figure.

- Short run equilibrium is determined by the intersection of $MPR = PC$.
- Long run equilibrium determined by $\pi^e = \pi^T = \pi_0$. 
With $\pi > \pi^T$, the Fed runs a recession ($x < 0$).

Note that $\pi < \pi^e$ so over time $\pi^e$ falls and this causes the $PC$ curve to shift down to restore equilibrium.
Fed stimulates the economy to offset the fall in inflation (\(\pi^T = 2\%\) by assumption)
New Policy Tradeoffs

Recall the slope of the $MPR$ curve is $-\left(\frac{\lambda}{k}\right)^{\frac{1}{\alpha}}$. Consider two economies with $\lambda_1 > \lambda_2$ (same $k$)

With positive inflation shock ($e > 0$), Economy 2 experiences greater fall in output but smaller inflation increase.

Lars Svensson of Princeton: Central Banks should tell us their $\lambda$!
New Policy Tradeoffs

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● Recall that in the bad old days (before rational expectations) - it was thought the policy tradeoff was between the level of inflation and the level of output.

● Now, in the new policy models, the policy tradeoffs are in terms of the implied changes in inflation and output - determined by the slope of the Monetary Policy Rule.
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- Scale this by full employment and use the Fisher relationship:

\[ \frac{y}{y^n} = \frac{y_0}{y^n} - b (i - \pi^e) + u \]  \hspace{1cm} (2)
Deriving an interest rate rule

Define the long run real interest rate when $y = y^n$ and $u = 0$. Use eq. (2)

$$\frac{y}{y^n} = \frac{y_0}{y^n} - b (i - \pi^e) + u$$

$$\left( \frac{y^n - y_0}{y^n} \right) = -br^* \implies r^* = \frac{1}{b} \left( \frac{y_0 - y^n}{y^n} \right) = \frac{1}{b} x_0 \tag{3}$$

Now subtract 1 from both sides of eq. (2) and rearrange terms

$$\frac{y - y^n}{y^n} = \frac{y_0 - y^n}{y^n} - b (i - \pi^e) + u = x_0 - b (i - \pi^e) + u$$

$$x = br^* - b (i - \pi^e) + u = -b \left( \frac{i - \pi^e - r^*}{r} \right) + u \tag{4}$$

If $r > r^* \implies x < 0$
Deriving an interest rate rule

Use our two previous key relationships to derive the reduced form model for $x$

$$\pi = \pi^e + ax + e \ (PC)$$

$$\pi = \pi^T - \alpha (x - u) \ (MPR)$$

Setting the two expressions equal and solving for $x$ yields (also, once again, we will ignore shocks - set $e = 0$):

$$x = \left( \frac{1}{a + \alpha} \right) (\pi^T - \pi^e) + \left( \frac{\alpha}{a + \alpha} \right) u$$

We will assume that the Fed can not respond to $u$ (unknown) - so drop from the equation to get the reduced form:

$$x = \left( \frac{1}{a + \alpha} \right) (\pi^T - \pi^e)$$
Deriving an interest rate rule

Recall the IS curve: \( x = -b(i - \pi^e - r^*) \). Since we are seeking the optimal setting for the nominal interest rate, solve for \( i \)

\[
i = -\frac{1}{b}x + \pi^e + r^*
\]

Since \( x = \left( \frac{1}{a+\alpha} \right) (\pi^T - \pi^e) \) this becomes

\[
i = -\frac{1}{b} \left[ \left( \frac{1}{a+\alpha} \right) (\pi^T - \pi^e) \right] + \pi^e + r^*
\]

or

\[
i = r^* + \pi^e + \frac{1}{b} \left( \frac{1}{a+\alpha} \right) \left( \pi^e - \pi^T \right)
\]
We have
\[ i = r^* + \pi^e + \frac{1}{b} \left( \frac{1}{a + \alpha} \right) (\pi^e - \pi^T) \]

Define the nominal interest rate target as \( i^T = r^* + \pi^T \). Introduce this into the above expression by adding and subtracting \( \pi^T \) to the RHS

\[ i = \left( r^* + \pi^T \right) + \left( \pi^e - \pi^T \right) + \frac{1}{b} \left( \frac{1}{a + \alpha} \right) (\pi^e - \pi^T) \]

Or, using the definition of the nominal interest rate target:

\[ i = i^T + \left[ 1 + \frac{1}{b} \left( \frac{1}{a + \alpha} \right) \right] (\pi^e - \pi^T) + \frac{e}{b (a + \alpha)} > 1 \]

The critical factor is that the nominal interest rate should move more than one-for-one with changes in expected inflation.
The Taylor rule

We have:

\[ i = i^T + \left[ 1 + \frac{1}{b} \left( \frac{1}{a + \alpha} \right) \right] (\pi^e - \pi^T) + \frac{e}{b(a + \alpha)} > 1 \]

If expected inflation is above the target inflation rate, then the nominal interest rate must increase by a greater amount to raise the real interest rate. This intuition is reflected in the famous Taylor rule that is used to characterize monetary policy.