1 Homework #4 - Answer Key

1. In his critique of economic policy analysis, Lucas derived the following demand curve for capital (i.e. investment) in a hypothetical industry:

\[ k_t (1 - \delta) + i_t = \frac{1}{\lambda} E_t (a_{t+1}) - \frac{b}{\lambda^2} \left[ \frac{r_t}{1 - \theta_t} + \delta \right] + \frac{b}{\lambda^2} \left[ \psi_t (1 + r_t) - E_t (\psi_{t+1}) (1 - \delta) \right] \]

where \( r_t \) denotes the current one-period interest rate, \( \theta_t \) is the current tax rate on profits, \( \psi_t \) is the investment tax credit. Answer the following:

(a) Explain why the factors on the right-hand side of eq. (1) affect investment demand.
(b) What two properties were used to derive this investment demand function.
(c) Lucas criticized Hall and Jorgenson’s analysis of the 1962 tax credit - where did they go wrong? The left-hand side (LHS) of the equation is the optimal desired capital stock of the firm.

ANSWER: (a) The first term on the right-hand side (RHS) reflects the state of future demand. Higher demand leads to output sold at a higher price and so firms would have a higher level of desired capital. The second term on the RHS is the effect of taxes, interest rates, and depreciation. Raising the profit tax reduces the profitability of investment and leads to lower desired capital stock. A higher interest rate raises the opportunity cost of capital and hence leads to a lower desired stock of capital. Greater depreciation leaves less capital leftover after production and hence less that can be sold (or a higher cost to maintain a level of production) and so also reduces the desired stock of capital. The third term on the RHS is the expected capital gain/loss. An expected rise in the investment tax credit tomorrow relative to today, lowers the expected capital gain today and hence lowers the desired level of the capital stock. (b) The solution assumes that firms are profit-maximizers that set marginal cost equal to expected marginal benefit. The solution also assumes that markets clear, i.e., markets are in equilibrium. (C) In Hall and Jorgenson’s analysis they treat the investment tax credit as permanent once it has been put in place. Lucas states that a more likely scenario is that an investment tax credit will be transitory (put in effect for a period of time and then eventually removed). Given that firms forecast the
behavior of policy rationally, they would react optimally in each of these situations. Lucas finds that the effect of an investment tax credit on the firm’s desired stock of capital is approximately 7 times as large with his set of assumptions relative to Hall and Jorgenson’s assumptions. This makes sense because under Lucas’s assumptions firm’s forecast that the policy is temporary and so there is a greater incentive to respond to the tax credit today relative to if they forecast that the policy is permanent as in Hall and Jorgenson.

2. Consider the following IS-LM model:

\[ Y = a_0 + a_1 r + u \]
\[ M = b_0 + b_1 Y + b_2 r + v \]

where \( u, v \) denote random shocks to the goods and money market respectively. Express the model in reduced form under the assumption that the money supply is the instrument of monetary policy.

ANSWER: Use the LM curve to express \( r \) as a function of \( M \), then substitute into the IS curve. This yields:

\[ Y = \frac{a_0 b_2 + b_2 u - a_1 v + a_1 (M - b_0)}{a_1 b_1 + b_2} \]

3. Blinder states (on p.27): “Returning to Poole’s dichotomy, let me remind you of his basic conclusion: that large LM shocks militate in favor of targeting interest rates while large IS shocks militate in favor of targeting the money supply.” Explain why Poole reached this conclusion.

ANSWER: If the economy is primarily hit by IS shocks (i.e. shocks to the goods market), then it is optimal to let the interest rate respond to these shocks. This response will stimulate investment (if a negative shock) or reduce investment (if a positive shock) and therefore help to stabilize output. If the shocks are primarily to the money market (LM curve), then the optimal policy is to insulate the real economy (output) from these shocks. This is done by using the interest rate as instrument. The reason central banks have adopted an interest rate target is that, in most modern economies, there is greater instability in the money market relative to the goods market.

4. Using the Poole model, prove that the optimal setting of the money supply (when \( M \) is the instrument) is given by:

\[ M^* = \frac{Y_f (a_1 b_1 + b_2) - a_0 b_2 + a_1 b_0}{a_1} \]

ANSWER: Use the reduced form for \( Y \) to eliminate \( Y \) in the loss function. Then the monetary authorities face the following minimization problem:

\[ \min_{M} \left[ \left( \frac{a_0 b_2 + b_2 u - a_1 v + a_1 (M - b_0)}{a_1 b_1 + b_2} - Y_f \right)^2 \right] \]
Taking the derivative and setting it equal to zero, we obtain:

\[ E \left[ \frac{2a_1}{a_1 b_1 + b_2} \left( \frac{a_0 b_2 + b_2 u - a_1 v + a_1 (M - b_0)}{a_1 b_1 + b_2} - Y_f \right) \right] = 0 \]

Taking expectations and solving for \( M \) yields:

\[ M^* = \frac{Y_f (a_1 b_1 + b_2) - a_0 b_2 + a_1 b_0}{a_1} \]

5. In the model of monetary policy by Walsh, he derives the monetary policy rule as:

\[ \pi = \pi^T - \alpha \left( x - u \right) \]

What determines the slope (\( \alpha \)) of this function? How does this term relate to the "new policy trade-offs" as described by John Taylor?

ANSWER: The slope of the monetary policy rule (MPR) is: \( \alpha = \left( \frac{1}{\lambda} \right) a \) where \( \lambda \) is the importance of output fluctuations in the policymaker's loss function and \( k \) is the importance of inflation volatility in the policymaker's preferences. The parameter \( a \) is the slope of the Phillips Curve. This relates to the new policy trade-offs since the slope of the MPR determines the relative volatility of inflation and output for a given shock to the Phillips Curve.

6. Estimates of the Taylor rule during the sample period 1960-1979 produce the following (ignoring constants) values:

\[ R_t = 0.813 (\pi_t - \pi^*) + 0.252 (y_t - \bar{y}_t) \]

What is the implication of these estimates? In particular, do they help to explain the U.S. economic experience during this time?

ANSWER: The Taylor Principle states that the coefficient on inflation in the Taylor rule should be greater than 1. This is required since this implies that monetary authorities raise the real interest rate when inflation is above the inflation target; a higher real interest rate reduces aggregate demand which will reduce inflation. In the above equation, the coefficient is less than one. Consequently, an increase in inflation results in a lower real interest rate; this, in turn, stimulates aggregate demand which will increase inflation further. Some economists think that this helps to explain the high inflation of the 1970's.